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UNITED STATES AIR FORCE

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Wright-Patterson Air Force Base, Ohio

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(6) MINIMUM TIME TURNS WITH THRUST REVERSAL.

(9) Master's THESIS,

AFIT/GAE/AA/79D-6

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AFIT/GAE/AA/79D-6

MINIMUM TIME TURNS WITH THRUST REVERSAL

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Graduate Aeronautical Engineering

December 1979

Acknowledgements

I would like to thank my thesis advisor, Captain Jim Rader, and my thesis committee consisting of Captain Jim Silverthorn and Professor John D'Azzo for their efforts in helping me complete this thesis. Without their help, I would have not obtained the results I did, nor would I have finished in time to graduate. I would also like to thank my wife, Deanna, for her support at home in helping me and enduring with me the struggle of completing AFIT.

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LIST OF SYMBOLS

x - distance (x-direction)

y - distance (y-direction)

h - altitude

V - velocity

θ - flight path angle

ψ - heading angle

S - wing area

W - weight

T - thrust

L - lift

D - drag

N - side force

ε - thrust angle of attack

ζ - thrust side slip angle

ϕ - bank angle

α - angle of attack

π - thrust control variable

g - gravitational acceleration

g_o - gravitational acceleration at sea level

ρ - density

ρ_o - density at sea level

σ - density ratio

τ - nondimensional time

C_L	-	lift coefficient
C_D	-	drag coefficient
C_{L_α}	-	lift curve slope
C_{D_o}	-	parasite drag coefficient
K	-	induced drag parameter
G	-	performance index (t_f)
F	-	augmented performance index
X	-	state vector
U	-	control vector
M	-	final condition vector
H	-	Variational Hamiltonian
λ	-	Lagrange multiplier vector (differential equations)
ν	-	Lagrange multiplier vector (final conditions)
A	-	unknown parameter vector
$(\ )_i$	-	initial value
$(\ )_f$	-	final value
$(\ )_c$	-	corner value
$(\ )_A$	-	$\frac{\partial(\ )}{\partial A}$ (A arbitrary)
$\dot{a}$	-	$\frac{da}{dt}$ (a arbitrary)
a^T	-	a transposed (a arbitrary)

Abstract

The object of this study is to find the optimal trajectories and corresponding minimum turning times for a high performance aircraft with and without thrust reversal to perform a prescribed turn, and then to compare those trajectories and times to evaluate the benefit of thrust reversal. Optimal control theory is applied to solve the minimum time to turn optimal control problem, using a suboptimal control problem approach and a second-order parameter-optimization method.

The results of the study found that the suboptimal control approach was effective in solving the problem, that thrust reversal is beneficial in reducing turning times if the aircraft's initial velocity is above the corner velocity, and that thrust reversal is not beneficial in performing a minimum time turn without losing energy.

MINIMUM TIME TURNS WITH THRUST REVERSAL

I Introduction

Background

The F-15 Eagle is an example of a high speed, high thrust, high performance aircraft. One of its notable characteristics of performance is having a thrust to weight ratio greater than unity. A great deal of research is presently being directed in the area of aircraft thrust. Not only in the sense of finding ways to increase thrust, but also to find ways to better use and manage thrust. For example, direct thrust is used in V/STOL aircraft such as the Harrier to decrease or eliminate take-off and landing distances. Jet engine exhaust is directed over the upper surface of the wing of Boeing's YC-14 to create more lift and thus enabling larger gross payload weights. Thrust vectoring through the use of special jet engine exhaust nozzles is being studied to determine if it can provide the directional stability and control to replace the verticle stabilizer of an F-111 aircraft. In-flight thrust reversal is another area of research, and is the one with which this thesis is concerned.

Thrust reversal has been used to a limited extent on both commercial and military aircraft for the purpose of decreasing landing roll. The question is, can thrust reversal improve the in-flight performance of a fighter aircraft like the F-15? Can it do so by increasing the aircraft's capability to decelerate? Thrust reversal may prove valuable during a high G turn to escape a pursuer or to close in on an advisary.

An Aircraft's turning performance is characterized by the time it takes to complete a turn. For a set of prescribed initial and final conditions describing a turn, there is an optimal trajectory and, thus, an optimal velocity profile that can be flown to complete the prescribed turn in minimum time. Thrust reversal could be beneficial in helping an aircraft follow that optimal velocity profile depending upon the initial conditions from which the turn is initiated. However, it could also decrease the aircraft's specific energy which would be unfavorable since the advantage in an air-to-air combat situation is maintained by the aircraft with the greatest specific energy.

Optimal control theory has been used extensively to find the optimal aircraft controls and thus, the optimal trajectories for a high performance aircraft without thrust reversal to perform a minimum time turn. However, no consideration has been given to the use of thrust reversal to improve the minimum time to turn. For this reason, thrust reversal is studied to determine its feasibility for use in improving an aircraft's turning performance.

Problem Statement

The problem is finding the optimal controls which will maneuver a high performance fighter aircraft through a particular turn prescribed by a set of initial and final conditions, and doing so in the minimum amount of time. The problem also includes finding those optimal controls and minimum turning times for an aircraft with and without thrust reversal. The turning maneuvers must also be within the aircraft's capability. For example, the aircraft's maximum angle of attack before buffeting and stalling occur cannot be exceeded. The maximum and minimum available thrust values cannot be exceeded. And last, the aircraft's structural load limit cannot be exceeded.

The object of this study is to find the optimal trajectories and corresponding minimum turning times for a high performance aircraft with and without thrust reversal to manuever a prescribed turn, and, then to compare those trajectories and turning times to evaluate the benefit of thrust reversal.

Scope

The scope of this study is limited to finding the optimal trajectories and minimum turning times for turns initiated from two different initial conditions, but always terminated by a single final condition. The scope also includes finding those trajectories and turning times for an aircraft with and without thrust reversal while all other aircraft characteristics remain the same.

Assumptions

The following assumptions are made in order to model the equations of motion, the aerodynamic and thrust forces, and the atmosphere: For the equations of motion, it is assumed that the aircraft's turns are performed over a flat earth with a constant gravitational acceleration, that the aircraft flies coordinated turns, that negligible fuel is consumed during the maneuvers, that the error due to the thrust vector not being colinear with the velocity vector is negligible, and that the error due to making small angle assumptions (i.e., $\cos\alpha=1$, $\sin\alpha=\alpha$, for small α) is also negligible. These assumptions are typical when describing the equations of motions for a point mass aircraft which is the model commonly used in studying minimum time turns for an aircraft. For the aerodynamic and thrust forces, it is assumed that the lift coefficient is a linear function of the angle of attack up to the maximum angle of attack before buffeting and stalling occur. The drag coefficient is a function of the square of the lift coefficient. These assumptions are based upon thin airfoil theory and are appropriate for use in this study. It is assumed that the maximum available thrust is constant throughout the turning maneuvers. This assumption is valid due to the small change in density ratio due to change in altitude during the turns. For the atmosphere it is assumed that the aircraft maneuvers are performed in the Standard Atmosphere is defined by NASA in Ref (1). This is common practice in the study of aircraft performance problems. It is also assumed that the aircraft initiates the turns from straight and horizontal flight, and that its angle of attack, bank angle, and thrust can change instantaneously. Instantaneous controls are valid since an exact model of control response is not necessary to study the benefit of thrust reversal.

All of these assumptions are made to both match the assumptions made in Ref (2) and to help simplify the complexity of the problem. The errors created due to these simplifications will be negligible and the stated purpose of this study can effectively be accomplished. The need to match assumptions with those in Ref (2) will be discussed in the approach to the problem.

Summary of Current Knowledge

Hennig, Bolding, and Helgeson, in Ref (2), obtained numerous results for minimum time turns for an aircraft without thrust reversal. Some results were obtained for turns initiated from two different initial conditions, but with the same final conditions, and for various maximum thrust to weight ratios. The results of two data sets (Ref [2:99]) are of importance for the sake of comparison. They are summarized as follows:

DATA SET	T/W	V_i (ft/sec)	h_i (ft)	V_f (ft/sec)	h_f (ft)	t_f (sec)
6	1.5	621	13,990	794	12,300	10.5
12	1.5	903	13,990	886	17,635	11.2

Table 1. Results from Previous Work (Ref [2])

These results are based upon the initial turning conditions of flight path angle, θ , and heading angle, ψ , both equal to zero, and the final turning conditions of $\theta_f=0$ and $\psi_f=180°$.

The same aircraft and the same initial and final conditions as specified in data sets 6 and 12 are used in this study for reasons discussed in the approach to the problem.

Approach

A particular aircraft and turning situation must be selected in order to study the benefit of thrust reversal upon turning performance. Also, a particular optimization technique must be employed to find the minimum time for the selected aircraft to perform the specified turn.

The particular aircraft and turning situations are selected from Ref (2). This is done for the sake of comparing results in Ref (2) to the results obtained in this study for an aircraft without thrust reversal. This also justifies the assumptions used as they are identical to those used in Ref (2). A comparison is necessary to lend credibility to the results of this study since a different optimization technique is used in this study.

The approach used to solve for the optimal controls and minimum turning times is developed by Hull and Edgeman in Ref (3). They develop a suboptimal control approach to optimal control problems using a second-order parameter optimization method. The problem of finding the optimal controls and minimum turning times is a very complex problem. The solution complications arise because the physics of the problem, such as describing the equations of motion for an aircraft, are so complex that an analytical solution is impossible. The optimal control problem approach is very complex and exhibits certain inherent solution difficulties. Ref (2) uses this approach with a gradient-restoration optimization technique. The inherent solution difficulties will be pointed out in section III. The suboptimal control problem approach reduces the complexity and eliminates the inherent solution difficulties associated with the optimal control problem as will be pointed out in section IV. It is for this reason that the suboptimal control approach is used.

The following sections will define the minimum time to turn problem, will analyze the optimal and suboptimal control problem approaches, will discuss solving the suboptimal control problem, and will present the results and conclusion obtained from the study.

II Defining the Minimum Time to Turn Problem

The first step in analyzing thrust reversal is to define the minimum time to turn problem. This involves defining the particular aircraft and the turn it is to perform, and then modeling the motion of the aircraft, the aerodynamic and thrust forces, and the atmosphere in which it flies. Also, the constraints upon the aircraft must be defined. Each of these items are now considered individually.

The Aircraft and the Turn

As mentioned previously, the aircraft and the type of turn performed are selected from Ref (2) for the purpose of comparing results. A hypothetical aircraft is used in Ref (2) and it is characterized by the following parameters (Ref [2:97]):

$C_{L_\alpha} = 5.0$ $\qquad$ $S = 237$ sq ft

$C_{D_o} = .02$ $\qquad$ $\alpha_{max} = .2$ radians

$K = .05$ $\qquad$ $(T/W)_{max} = 1.5$

$W = 12{,}150$ lb. $\qquad$ $(L/W)_{max} = 7.22$

Some of these parameters will be used when modeling the aerodynamic and thrust forces. The type of turn performed assumes the aircraft to be flying straight and horizontal immediately before entering the turn. Thus, the initial heading and flight path angles equal zero. The turn is completed when $\theta_f = 0$ and $\psi_f = 180°$. Thus, the initial and final conditions of the problem are as follows:

CASE	V_i (ft/sec)	h_i (ft)	θ_i°	ψ_i°	θ_f°	ψ_f°
1	621	13,990	0	0	0	180
2	903	13,990	0	0	0	180

Table 2. Initial and Final Conditions of the Turns

Since in both cases the initial altitude is 13,990 ft, the gravitational acceleration at that altitude, $g=32.131$ ft/sec^2 (Ref [1:160]) is used as the constant gravitational acceleration throughout a turn.

Modeling the Motion of the Aircraft

The equations of motion for flight of a point mass aircraft over a flat earth are derived in Ref (4:48-49) and are:

$$\dot{x} = V\cos\theta\cos\psi \tag{1}$$

$$\dot{y} = V\cos\theta\sin\psi \tag{2}$$

$$\dot{h} = V\sin\theta \tag{3}$$

$$\dot{V} = g\left(\frac{T}{W}\cos\varepsilon\cos\zeta - \frac{D}{W} - \sin\theta\right) \tag{4}$$

$$\dot{\theta} = \frac{Qg\sin\phi}{WV} + \frac{gL\cos\phi}{WV} - \frac{g\cos\theta}{V} - \frac{Tg}{WV}(\sin\phi\cos\varepsilon\sin\zeta - \cos\phi\sin\varepsilon) \tag{5}$$

$$\dot{\psi} = \frac{g\sin\phi}{V\cos\theta}\left(\frac{T}{W}\sin\varepsilon + \frac{L}{W}\right) \tag{6}$$

Assuming that a coordinated turn is performed, that the error due to the thrust vector not being colinear with the velocity vector is negligible, and that the error due to making small angle assumptions is negligible, then $Q=0$, $\sin\zeta=0$ and $\varepsilon=\alpha$, and $\sin\alpha=\alpha$ and $\cos\alpha=1$, respectively.

The equations of motion then become:

$$\dot{x} = V\cos\theta\cos\psi \tag{7}$$

$$\dot{y} = V\cos\theta\sin\psi \tag{8}$$

$$\dot{h} = V\sin\theta \tag{9}$$

$$\dot{V} = g(\frac{T}{W} - \frac{D}{W} - \sin\theta) \tag{10}$$

$$\dot{\theta} = \frac{g}{V}[(\frac{T}{W}\alpha + \frac{L}{W})\cos\phi - \cos\theta] \tag{11}$$

$$\dot{\psi} = \frac{g\sin\phi}{V\cos\theta}(\frac{T}{W}\alpha + \frac{L}{W}) \tag{12}$$

These equations are used to model the motion of the aircraft with respect to an earth fixed coordinate frame. The state variables in these equations are x , y , h , V , θ , and ψ . The control variables are α , ϕ , and T . A new control variable for T will be defined in the following discussion.

The Aerodynamic/Thrust Forces

Lift and drag are the two aerodynamic forces in Eqs (10) through (12) and can be expressed as:

$$L = \frac{\rho_o \sigma V^2 S C_L}{2} \tag{12}$$

$$D = \frac{\rho_o \sigma V^2 S C_D}{2} \tag{13}$$

The assumptions made based upon thin airfoil theory lead to the following expressions for the lift and drag coefficients:

$$C_L = C_{L_\alpha} \alpha \tag{14}$$

$$C_D = C_{D_o} + KC_L^2 \tag{15}$$

For use in the equations of motions, the expressions for $\frac{L}{W}$ and $\frac{D}{W}$ are:

$$\frac{L}{W} = \frac{\rho_o \sigma V^2 S}{2W} C_{L_\alpha} \alpha \tag{16}$$

$$\frac{D}{W} = \frac{\rho_o \sigma V^2 S}{2W} (C_{D_o} + KC_L^2) \tag{17}$$

The thrust to weight ratio is also needed for use in the equations of motion. Since it is assumed the maximum thrust is constant during turns, thrust will be defined by:

$$T = T_{max} \pi \tag{18}$$

where π is the thrust control variable. Assuming constant aircraft weight during turns, the thrust to weight ratio becomes:

$$\frac{T}{W} = \frac{T_{max}}{W} \pi = (\frac{T}{W})_{max} \pi \tag{19}$$

<u>The Atmosphere</u>

Assuming a standard atmosphere, the density ratio (Ref [5]) is expressed as:

$$\sigma = \frac{\rho}{\rho_o} = [1 - (\frac{n-1}{n}) \frac{g_o}{RT_o} h]^{\frac{1}{n-1}} \tag{20}$$

where

$$\rho_o = .002378 \text{ slugs/ft}^3$$

$$g_o = 32.174 \text{ ft/sec}^2$$

$$T_o = 518.688° \text{ R}$$

$$n = 1.235$$

$$R = 1715 \text{ ft}^2\text{/sec}^2\text{-°R}$$

The Control Variable Constraints

There are three physical constraints upon the aircraft. The aircraft has a maximum angle of attack, a maximum structural load limit or load factor, and a maximum and minimum thrust. These constraints are expressed as:

$$\alpha \leq \alpha_{max} \tag{21}$$

$$\frac{L}{W} \leq \left(\frac{L}{W}\right)_{max} \tag{22}$$

and

$$T_{min} \leq T \leq T_{max} \tag{23}$$

or, since a new control variable for thrust is defined

$$\pi_{min} \leq \pi \leq \pi_{max} \tag{24}$$

The load factor constraint will be discussed more fully in section V.

III The Optimal Control Problem Approach

The optimal control problem approach is discussed to show the complexity and the inherent solution difficulties of the approach. It is also valuable to do so as some important information is learned concerning the nature of the solution. First, the minimum time to turn optimal control problem is defined. Second, the conditions to be satisfied by a solution are discussed. And third, the method of solving for a solution is discussed.

The Optimal Control Problem

The optimal control problem is a functional minimization problem. It is desired to find the functional relationships for the control variables that will minimize the time to turn, or the performance index

$$G = t_f \tag{25}$$

subject to the differential constraints (equations of motion) expressed vectorally as

$$\dot{X} = f \tag{26}$$

where X is the state vector containing the state variables and f is the vector containing the right side of Eqs (7) through (12). The problem is also subject to the control variable constraints, Eqs (21) through (24), and the initial and final conditions, Table ?. The final conditions are expressed vectorally as M. That is

$$M_1 = \theta_f \tag{27}$$

$$M_2 = \psi_f - 180° \tag{28}$$

If the final conditions are satisfied, then

$$M = 0 \tag{29}$$

The Calculus of Variations is applied to find the necessary and sufficient conditions to be satisfied by the optimal control variables.

<u>Conditions to be Satisfied</u>
(Ref [6:149-154])

If the augmented performance index, J, is defined by

$$J = G + \nu^T M + \int_{t_i}^{t_f} \lambda^T (f - \dot{X})\, dt \tag{30}$$

where ν and λ are Lagrange multiplier vectors, then the Variational Hamiltonian, H, is expressed vectorally as

$$H = \lambda^T f \tag{31}$$

The optimal controls expressed vectorally as U must satisfy the first variational requirement or the Euler-Lagrange equations expressed vectorally as

$$\dot{\lambda} = -H_X{}^T \tag{32}$$

$$\dot{X} = H_\lambda{}^T \tag{33}$$

$$U_{OPT} = \min_{U} H \tag{34}$$

Eq (34) basically states that the optimal controls are those that minimize H. The first variation requirement also provides what are termed natural or transversality conditions and corner conditions expressed vectorally as:

$$H_i = G_{t_i} = 0 \tag{35}$$

$$H_f + G_{t_f} = 0 \tag{36}$$

$$\lambda_i^T + G_{X_i} = 0 \tag{37}$$

$$\lambda_f^T - G_{X_f} = 0 \tag{38}$$

$$\Delta(H) - G_{t_c} = 0 \tag{39}$$

$$\Delta(\lambda^T) + G_{X_c} = 0 \tag{40}$$

$$G_{\nu} = 0$$

As can be seen, the complexity of the problem is substantial and becomes even more complex when attempting to solve to the problem.

Solving the Optimal Control Problem (Ref [7])

The optimal control problem, when the controls are neither on the boundaries nor singular, can be easily transformed into a boundary value problem. In this case the solution for Eq (34) is identical to solving $H_U=0$ for the control variables as functions of X and λ.

These expressions for the control variables are substituted into the $\dot{\lambda}$ and $\dot{X}$ equations. By guessing the unknown initial values, the $\dot{\lambda}$ and $\dot{X}$ equations can then be numerically integrated from the initial time to the guessed final time and the resulting final conditions compared with the desired results. Various optimizing techniques can be employed to iteratively change the guesses for the unknown values and attempt to satisfy Eqs (32) through (42).

Inherent solution difficulties are involved in guessing the final time and the unknown initial values for the λ 's. Guessing the final time is easier since it is a physical variable in the problem. But this is not so for the λ 's since they are not physical variables and one has no intuitive feeling for their values. And, if there are any corner conditions, they must satisfy Eqs (40) through (41). Also, guesses for unknown corner values must be made. Many of the optimization techniques that can be used require good initial guesses for the unknown values before the technique will converge. Other optimizations techniques do not require good initial guesses; however, they are slow to converge. Thus, guessing the unknowns makes finding a solution very difficult.

Another solution difficulty, present in the problem, occurs because one of the control variables is a singular control. This can be demonstrated by looking at the second partial derivatives of the Variational Hamiltonian. The Variational Hamiltonian expanded from vector notation becomes

$$H = \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3 + \lambda_4 f_4 + \lambda_5 f_5 + \lambda_6 f_6 \qquad (42)$$

where the f 's are the right sides of Eqs (7) through (12). The perfect derivative of the Hamiltonian with respect to T is

$$H_T = \lambda_4 \frac{g}{W} + \lambda_5 \frac{g\alpha}{VW} \cos\phi + \lambda_6 \frac{g\alpha \sin\phi}{VW\cos\theta} \tag{43}$$

The second partial derivative is

$$H_{TT} \equiv 0 \tag{44}$$

Eq (44) identifies the thrust control as a singular control. The $H_{\phi\phi}$ and $H_{\alpha\alpha}$ equations are not identically equal to zero. Eq (43) is not a function of T ; therefore, H is linear with respect to T and H_T is the value of the slope. Thus, in order to minimize H with respect to T , thrust must satisfy one of three solutions depending upon the value for H_T . These solutions are:

1. $T = T_{min}$ when $H_T > 0$
2. $T = T_{max}$ when $H_T < 0$
3. $T_{min} \leq T \leq T_{max}$ when $H_T = 0$ (45)

The problem can still be transformed into a boundary value problem but not as easily. In order to find an expression for T when $H_T=0$, one must use the relationship

$$\frac{d^n(H_T)}{dt^n} = 0 \qquad (n = 1,2,3\ldots) \tag{46}$$

starting with $n=1,2,\ldots$ until an expression containing T can be obtained. Then when $H_T=0$, that expression can be substituted into the $\dot{X}$ and $\dot{\lambda}$ equations, etc.

Thus, it is known that the thrust is a singular control and that it adds to the difficulty of finding a solution. It is therefore desirable to use a different approach which simplifies the complexity of the problem and which can handle singular controls more easily. The suboptimal control approach accomplishes that task.

IV The Suboptimal Control Problem Approach
(Ref [7] and [3])

The suboptimal control problem approach is discussed to show how it simplifies the optimal control problem and eliminates the inherent solution difficulties involved with the optimal control problem approach. First, the minimum time to turn suboptimal control problem is defined. Second, the conditions to be satisfied by a solution are discussed. And third, the method of solving the suboptimal control problem is discussed.

The Suboptimal Control Problem

The suboptimal control problem is a parameter optimization problem, but otherwise it is basically the same as the optimal control problem. The main difference is that in the suboptimal control problem the control variables are described by some mathematical form with unknown coefficients defined by vector B . The mathematical form expressed functionally as

$$U = U(t,B) \tag{47}$$

can be an ordinary polynomial, a Fourier series, a Chebyshev series, etc. If A denotes the following vector of unknown parameters:

$$A = [t_f, B]^T \tag{48}$$

then the Eqs (7) through (12) can be integrated from $t=0$ to $t=t_f$, subject to all control variable constraints, and the resulting final conditions expressed functionally will be

$$X_f = X_f(A) \tag{49}$$

Or in other words, the final states are a function of the parameter vector A . And thus so are the performance index and the final condition vector M .

Hence, the minimum time to turn problem expressed functionally entails finding the parameter vector A which minimizes the time to turn, or the performance index

$$G = G(A) \tag{50}$$

subject to the differential constraints (equations of motion)

$$\dot{X} = f(X,A,t) \tag{51}$$

the control variable constraints, the initial conditions, and the final condition vector

$$M(A) = 0 \tag{52}$$

Ordinary Calculus is applied to find the necessary conditions to be satisfied by the solution for the parameter vector A .

Conditions to be Satisfied

If the augmented performance index, F , is defined functionally as:

$$F(A,\nu) = G(A) + \nu^T M(A) \tag{53}$$

an optimal parameter vector A must satisfy the first variational requirements expressed functionally as

$$F_A^T(A,\nu) = 0 \tag{54}$$

$$M(A) = 0 \tag{55}$$

where

$$F_A = \frac{\partial F}{\partial A} \tag{56}$$

and contains what is termed first-order information or information concerning the change in F with respect to changes in the elements in A. Thus, F_A represents a slope and indicates in which direction A should change to drive F_A towards zero. F_A^T is a column vector containing elements equal in number to the number of unknown parameters in the vector A. M is the final condition matrix containing two elements. These are the <u>only</u> requirements a solution must satisfy. Thus, the suboptimal control problem is extremely less complex and does not have the inherent solution difficulties associated with the optimal control problem. If an effective method of solving for an optimal parameter vector A is available, then the suboptimal control approach is a very desirable means to study thrust reversal.

Method of Solution

Hull and Edgeman in Ref (3) formulate a second-order parameter-optimization technique and algorithm specifically for application to suboptimal control problems. In summary, the technique uses second-order information to determine how to change the parameters in the vector A

and the Lagrange multiplier vector ν such that the vectors F_A^T and M are driven to zero. The vector relationships used to change A and ν are:

$$\delta\nu = (M_A F_{AA}^{-1} M_A^T)^{-1} \; (-PM_A F_{AA}^{-1} F_A^T + QM) \tag{57}$$

$$\delta A = -F_{AA}^{-1} (PF_A^T + M_A^T \delta\nu) \tag{58}$$

where

$$M_A = \frac{\partial M}{\partial A} \tag{59}$$

$$F_{AA} = \frac{\partial^2 F}{\partial A^2} \tag{60}$$

and P and Q are scaling factors which control optimization and end condition satisfaction, respectively. F_{AA} contains the second-order information mentioned above. F_{AA} represents a change in slope and indicates the direction in which slope is increasing or decreasing. The algorithm developed to iteratively change A and ν goes as follows:

1. guess A and ν ;
2. integrate the equations of motions to obtain X_f ;
3. compute M , M_A , M_{AA} , F_A , and F_{AA} ;
4. select values for P and Q and compute $\delta\nu$ and $\delta\alpha$;
5. set $A=A+\delta\nu$ and $\nu=\nu+\delta\nu$;
6. check convergence criteria and if unsatisfied go to step 2.

This algorithm lends itself for use in developing a computer program to speed computations and thus, quickly find an optimal parameter vector A . The procedure is straightforward; however, some discussion on guessing ν , selecting P and Q , and stopping the computation is required.

By using a gradient or first-order approach, an initial value for ν can be computed, thus eliminating the need to guess ν. Hull and Edgeman do so and obtain the following vector relationships:

$$\nu = (M_A M_A^T)^{-1} [(Q/P)M - M_A G_A^T] \tag{61}$$

$$\delta A = -PF_A^T \tag{62}$$

where

$$G_A = \frac{\partial G}{\partial A} \tag{63}$$

F_A contains the first-order information used to calculate δA. In practice Eqs (61) through (62) should be employed initially for a number of iterations until they become inefficient. Then, with the current values for ν and A, switch to the second-order equations (Eqs [57] through [58]).

The procedure for selecting P and Q is not obvious. P and Q must be selected to insure that $\delta\nu$ and δA do not become too large so that ν and A are not changed too much such that the algorithm will not converge to $F_A=0$ and $M=0$. When in the neighborhood of the optimum A, P and Q can be chosen so that the norms $||F_A^T||$ and $||M||$ always decrease. A search on P and Q can be conducted to find the lowest values for $||F_A^T||$ and $||M||$. P and Q can be chosen on the basis that $||\delta\nu||$ and $||\delta A||$ are to be no greater than certain percentages of $||\nu||$ and $||A||$, respectively. Finally, the iterative procedure can be started with small values of P and Q; and, when it is apparent that the optimal solution is being approached, their values can be increased. Regardless of the approach followed, it is necessary that $P=1$ and $Q=1$ when in the final stages of convergence. If the second-order method is started with the results obtained from the gradient method which satisfy the final conditions, one should set $Q=1$ to hold the final conditions while optimizing.

Convergence is achieved when $||F_A^T||$ and $||M||$ are less than some small positive quantity, since it is desired to satisfy the requirements $F_A^T=0$ and $M=0$. An alternate approach is to monitor $||\delta\nu||$ and $||\delta A||$ and when they are less than some small positive quantity, terminate the algorithm.

V Solving the Minimum Time to Turn Suboptimal Control Problem

The minimum time to turn suboptimal control problem is sufficiently complex to require numerical methods and the use of a computer to seek solutions. In section II the minimum time to turn problem was defined and all the aircraft parameters, turning situations, equations of motion, aerodynamic/thrust forces, atmospheric parameters, and control constraints were specified. It is now necessary to adapt the defined problem for application in the algorithm specified in section IV. This will include deriving the final form of the equations of motion, incorporating the control variable constraints into the problem, and defining the mathematical form for the control variables so that the equations of motion may be integrated to find X_f . Next, the numerical methods to be used to find X_f , M , M_A , M_{AA} , F_A , and F_{AA} must be specified. And last, convergence criteria and an approach to guessing A must be determined. These areas are discussed individually as follows.

Equations of Motion

Eqs (7) through (12) are to be integrated to find X_f . The final time is one of the unknown parameters in the problem. It will be convenient to define a nondimensional time as

$$\tau = \frac{t}{t_f} \qquad (0 \le \tau \le 1) \tag{64}$$

so that the equations of motion are integrated from $\tau=0$ to $\tau=1$ regardless of the final time parameter. The equations of motion must be transformed so that they will be functions of the nondimensional time τ. By the chain rule

$$\dot{X} = \frac{dX}{dt} = \frac{dX}{d\tau}\frac{d\tau}{dt} = \frac{dX}{d\tau}\left(\frac{1}{t_f}\right) \tag{65}$$

and thus

$$\frac{dX}{d\tau} = t_f\,\dot{X} \tag{66}$$

and Eqs (7) through (12) can be transformed by simply multiplying the right side by the final time. By doing this and substituting the known aircraft and atmospheric parameters into Eqs (16), (17), (19), and (20), and substituting the constant gravitational acceleration and Eqs (16), (17), (19), and (20) into Eqs (10) through (12), the resulting equations of motion as a function of τ are:

$$\frac{dX}{d\tau} = t_f V\cos\theta\cos\psi \tag{67}$$

$$\frac{dy}{d\tau} = t_f V\cos\theta\sin\psi \tag{68}$$

$$\frac{dh}{d\tau} = t_f V\sin\theta \tag{69}$$

$$\frac{dV}{d\tau} = t_f\{48.1965\pi - V^2[(.0000149 + .0009315\,\alpha^2)(1 - .0000069h)^{4.256}] - 32.131\,\sin\theta\} \tag{70}$$

$$\frac{d\theta}{d\tau} = t_f\left(\frac{32.131\ \sin\phi}{V}\right)[(1.5\pi\alpha$$

$$+ .000116V^2\alpha(1 - .0000069h)^{4.256})\cos\phi - \cos\theta] \tag{71}$$

$$\frac{d\psi}{d\tau} = t_f\left(\frac{32.131\ \sin\phi}{V\cos\theta}\right)(1.5\pi\alpha$$

$$+ .000116V^2\alpha(1 - .0000069h)^{4.256}) \tag{72}$$

<u>The Control Variable Constraints</u>

Eq (24) defines the thrust control variable constraint. The thrust control variable π has a maximum value of one. Its minimum value is zero for an aircraft with no thrust reversal. Thus, the thrust control variable constraint for no thrust reversal becomes

$$0 \leq \pi \leq 1.0 \tag{73}$$

For an aircraft with thrust reversal, the minimum π value is specified to be $-.6$. This value is selected based upon educated estimates of the maximum reverse thrust capable and that which could be sustained by a pilot during the turn. Thus, the thrust control variable constraint for thrust reversal becomes

$$-.6 \leq \pi \leq 1.0 \tag{74}$$

Eqs (21) and (22) both define a constraint upon the angle of attack. The load factor constraint becomes

$$\frac{\rho_o \sigma V^2 S C_{L_\alpha} \alpha}{2W} \leq \left(\frac{L}{W}\right)_{max} \tag{75}$$

when Eq (12) is substituted into Eq (22). Substituting the known parameters into Eq (76) and (21) the load factor constraint can be reduced to

$$\sigma V^2 \alpha \leq 62660.6 \tag{76}$$

and the angle of attack constraint becomes

$$\alpha \leq .2 \tag{77}$$

These two constraints can be seen pictorially in Fig. 1.

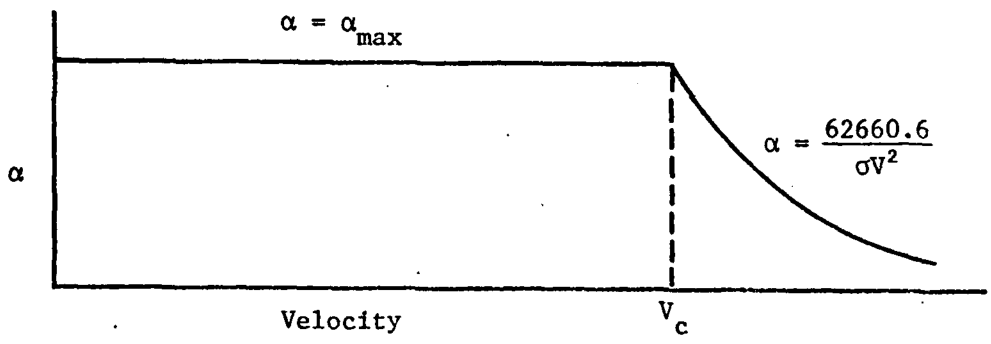

Fig 1. Angle of Attack vs Velocity Constraint

The corner velocity V_c is the velocity at which the lift coefficient required for flight at maximum load factor is equal to the maximum lift coefficient. Based upon this definition the corner velocity can be expressed as

$$V_c = \left(\frac{62660.6}{\alpha_{max}\sigma}\right)^{\frac{1}{2}} \tag{78}$$

Substituting in for α_{max}, the corner velocity relationship becomes

$$V_c = 559.735\ \sigma^{-\frac{1}{2}} \tag{79}$$

This relationship becomes important when studying the optimal trajectories which result in minimum times.

The control variable constraints are incorporated into the problem basically by setting the control variable equal to its maximum value when it exceeds its maximum and equal to its minimum value when it falls below its minimum value. For higher order controls, such as linear, quadratic, etc., the control histories may intersect their boundaries at various times which can be predetermined before integrating the equations of motion. If this situation occurs integration takes place from $\tau=0$ to $\tau=\tau_1$, from $\tau=\tau_1$ to $\tau=\tau_2$, etc., where τ_1, τ_2, etc., are the times, ordered consecutively, when the controls intersect their boundaries. Doing this insures accuracy when integrating the equations of motion such that the value for the controls change exactly at the correct time during integration.

For the load factor constraint it cannot be predetermined when the aircraft's velocity will equal the corner velocity. When the aircraft's velocity is below the corner velocity α can equal the value expressed by the mathematical form describing the angle of attack so long as α_{max} is not exceeded. When the aircraft's velocity is above the corner velocity α_{max} must be described by

$$\alpha_{max} = \frac{62660.6}{V^2\sigma} \tag{80}$$

This is accomplished by checking during integration to determine when the corner velocity is reached and changing to the appropriate describing equation for angle of attack when it is.

Mathematical Form for Describing the Control Variables

The mathematical form used to describe the control variables is a series using Chebyshev polynomials defined on the interval $(0,1)$. The polynomials, T_j, as a function of τ for $j=1$ to $j=6$ are

$$T_1 = 1 \tag{81}$$

$$T_2 = 2\tau - 1 \tag{82}$$

$$T_3 = 8\tau^2 - 8\tau + 1 \tag{83}$$

$$T_4 = 32\tau^3 - 48\tau^2 + 18\tau - 1 \tag{84}$$

$$T_5 = 128\tau^4 - 256\tau^3 + 160\tau^2 - 32\tau + 1 \tag{85}$$

$$T_6 = 512\tau^5 - 1280\tau^4 + 1120\tau^3 - 400\tau^2 + 50\tau - 1 \tag{86}$$

The control variables are then described by

$$\phi = \sum_{\ell=1}^{NPH} B_\ell T_\ell \tag{87}$$

$$\pi = \sum_{m=1}^{NPI} C_m T_m \tag{88}$$

$$\alpha = \sum_{n=1}^{NA} D_n T_n \tag{89}$$

where B , C , and D are the unknown coefficients with NPH , NPI , and NA the number of unknown coefficients for ϕ , π , and α respectively.

It is appropriate here to talk about the conditions under which the α and π coefficients will not be parameters in the problem. Take the case when α and π are described with only one coefficient, or are constant controls. If α and/or π are at their maximum values and if the F_A terms corresponding to the α and/or π coefficients are negative, then in order to change the coefficients so as to drive their corresponding F_A terms to zero (recall $F_A=0$ is desired), the α and/or π coefficients must exceed their allowable maximum value. Vice versa, if π is at its minimum value and F_A corresponding to the π coefficient is positive, then the π coefficient must fall below its allowable minimum value. In either case, the α and/or π coefficients no longer become parameters because their values are fixed at their maximum or minimum limits and their corresponding F_A terms cannot be driven to zero. In such a case the effect of the α and/or π coefficients must be eliminated in order to calculate $\delta\nu$ and δA without their influence. This is accomplished by dropping all references to these parameters in the matrices used to calculate $\delta\nu$ and δA .

Numerical Methods

Eqs (57), (58), (61), and (62) contain the matrices M , M_A , M_{AA} , F_A , and F_{AA} . These matrices are determined using numerical techniques. F_A and F_{AA} can be evaluated from the expressions (Ref [3:484]):

$$F_A = G_A + \nu^T M_A \tag{90}$$

$$F_{AA} = G_{AA} + \nu_1 M_{AA1} + \nu_2 M_{AA2} \tag{91}$$

where G_A is known analytically and $G_{AA}=0$ since $G=t_f$. Thus, the only unknowns are M , M_A , and M_{AA} .

To evaluate M , Eqs (67) through (72) must be integrated to determine the final states. A fifth-order, Runge-Kutta, controllable step size integration technique using Fehlberg coefficients (Ref [8]) is used for the integration. The Fehlberg technique is used because it controls the amount of truncation error in each integration step by controlling the step size. Thus, it can take the largest step possible without violating the allowable truncation error prescribed in the program. The result is fewer steps taken to integrate and thus, smaller accumulated truncation error. This makes the Fehlberg technique more efficient and accurate than other Runge-Kutta techniques. This proves beneficial when calculating numerical derivatives.

The M_A and M_{AA} matrices are determined using a central differences numerical derivative technique (Ref [9:21-22]). Using a nominal A_n where A_n are the individual elements in A , the equations of motion are integrated to obtain a nominal M , then using a positively perturbed A_n or

$$A_{n+} = A_n + \delta_n \tag{92}$$

and a negatively perturbed A_n or

$$A_{n-} = A_n - \delta_n \tag{93}$$

where δ_n is some small positive value, M_+ and M_- are obtained. The central differences representation for M_{A_n} is

$$M_{A_n} = \frac{M_+ - M_-}{2\delta_n} + \sigma(\delta_n^2) \tag{94}$$

where $\sigma(\delta_n^2)$ represents an error term of order of magnitude δ_n^2 . The M_A matrix contains two rows. The first row is determined using M_1 values in Eq (94) and the second row is determined using M_2 values in Eq (94). The M_{AA} matrices are determined in a similar manner; however, two elements A_n and A_m *must be preturbed both positively* and negatively to obtain M_{++} , M_{--} , M_{+-} , and M_{-+} . The central differences representation for $M_{A_nA_m}$ are

$$M_{A_nA_m} = \frac{M_+ - 2M + M_-}{\delta_n^2} + \sigma(\delta_n^2) \tag{95}$$

if n=m , and

$$M_{A_nA_m} = \frac{M_{++} - M_{+-} - M_{-+} + M_{--}}{4\delta_n\delta_m} + \sigma(\delta_n\delta_m) \tag{96}$$

if $n \neq m$. The M_{AA} matrix is actually two matrices. One is determined by using M_1 values in Eqs (95) and (96) and the other by using M_2 values in Eqs (95) and (96).

The error terms in Eqs (94) through (96) can be ignored if δ_n and δ_m are small enough. The best accuracy in using the central difference representations is obtained by using the smallest δ possible. Caution must be taken when selecting δ so that the numerator of Eqs (94) through (96) is not of the same order of magnitude as the round-off or truncation error associated with M due to the integration routine. The δ used for the central difference technique is

$$\delta_n = (\text{DELTA})A_n \tag{97}$$

and if the absolute value of δ_n is larger than DELTA then

$$\delta_n = \text{DELTA} \tag{98}$$

where DELTA is some small positive number. Thus, δ_n is controlled by the value used for DELTA . To select the best DELTA to use several calculations of M_A and M_{AA} were made by varying values for DELTA . By observing the variation between values of M_A and M_{AA} using different DELTA 's varied by a factor of ten from 10^{-1} to 10^{-7} , it was observed that for

$$\text{DELTA} \geq 10^{-2} \tag{99}$$

significant differences in M_A 's and M_{AA} 's resulted due to the error terms in Eqs (94) through (96) becoming too large. Thus, δ was too large. For

$$\text{DELTA} \leq 10^{-6} \tag{100}$$

differences began to be significant meaning round-off error due to the integration routine was the same order of magnitude as the numerator terms in Eqs (94) through (96). For

$$10^{-5} \leq \text{DELTA} \leq 10^{-3} \tag{101}$$

the least variation occurred and DELTA=10^{-4} was selected as the best value to use.

Convergence Criteria

Convergence is controlled by the scaling factors P and Q. When Eqs (61) through (62) were used to start the iterative process Q was set equal to P and P was selected small and was gradually increased as convergence progressed. When this process became inefficient and Eqs (57) through (58) were used Q was set equal to one and P was selected small and was gradually increased as convergence progressed.

The convergence criteria used to stop the second-order process once P became equal to one was

$$||M|| \leq 10^{-4} \tag{102}$$

$$||F_A|| < 10^{-4} \tag{103}$$

where $||F_A||$ was calculated only using the F_A terms which were not associated with a boundary control as described earlier.

Approach to Guessing the Controls

Since lift is what an aircraft uses to turn itself it is expected that the aircraft can turn fastest by generating the largest amount of lift possible. In order to do so the aircraft's angle of attack should be on the boundary as shown pictorially in Fig. 1. It is also expected that initially the aircraft's thrust parameter will be at its maximum or minimum value depending from which initial conditions the turn is initiated. This is expected also because it is known that thrust is a singular control.

Based upon these expectations, the approach to guessing the controls was to initially guess each control to be a constant value. The algorithm is free to change that constant to any constant value within the constraints of the problem. Once a solution for constant controls was obtained, then the bank angle control was changed to a linear control by adding another coefficient to the Chebyshev series describing the bank angle while the thrust and angle of attack controls continued to be expressed as constant controls. This procedure was continued until adding another coefficient to the bank angle control produced little change in the time to turn. Once these results were obtained and analyzed, then other higher order controls for thrust and angle of attack were considered and used to improve turning times.

This approach was used and results were obtained initially for three cases. Case 1 was for the prescribed aircraft without thrust reversal initiating the turn from an initial velocity below the corner velocity or V_1=621 ft/sec (V_c @ 13990 ft = 694 ft/sec). Case 2 was for the prescribed aircraft without thrust reversal initiating the turn from an initial velocity above the corner velocity or V_i=903 ft/sec . Case 3 was for the prescribed aircraft with thrust reversal initiating the turn from an initial velocity above the corner velocity. All of these cases used various orders of bank angle controls and constant thrust and angle of attack controls. Then results for three additional cases, Case 4, 5, and 6, were obtained using higher-order thrust controls in Case 1, 2, and 3, respectively: These results are presented in the following section.

VI Results

It is appropriate to discuss the results obtained for the optimal angle of attack control. A constant form of expressing angle of attack when the aircraft's velocity was below the corner velocity was used in all cases. The constant coefficient was free to be any value less than or equal to 0.2 . In all cases, the optimal coefficient value was the maximum allowable value, and the associated F_A term was negative indicating that the coefficient desired to be greater than its maximum value. It was anticipated that if an optimal angle of attack history existed that was other than the fixed maximum allowable limit, a lower constant coefficient would be found which would curve fit that optimal control history similar to the way the constant bank angle control fit the higher-order bank angle controls found. This situation never occurred and all the following results presented found the optimal angle of attack control to be its maximum value.

The results for Case 1 are listed and shown in Table 3, Fig. 2, and Fig. 3. Table 3 lists the optimal coefficients found for the various forms of the controls used and the resulting Lagrange multipliers. Fig. 2 shows the various optimal bank angle histories and lists the associated optimal constant thrust control variable values for each type of bank angle history. Fig. 3 shows the aircraft's trajectories plotted as altitude vs velocity for each of the various bank angle controls and also shows those trajectories in relation to the corner velocity curve labeled V_c . The results for Cases 2 and 3 are similarily listed and shown in Table 4, Fig. 4, Fig. 5, and Table 5,

Table 3

Optimal Coefficients for Case 1

	A	CONSTANT	LINEAR	QUADRATIC	CUBIC	QUARTIC	QUINTIC
MINIMUM TIME	t_f	.9643169E+01	.9636037E+01	.9617625E+01	.9582857E+01	.9579141E+01	.9575270E+01
BANK ANGLE COEFFICIENTS	B_1	.1425493E+01	.1426686E+01	.1470458E+01	.1487058E+01	.1478811E+01	.1478629E+01
	B_2		.4986949E+00	.5492497E+00	.1042854E+01	.1046252E+01	.1066866E+01
	B_3			.1179553E+00	.1231640E+00	.9953951E-01	.1010573E+00
	B_4				-.2277185E+00	-.2309636E+00	-.2052865E+00
	B_5					-.4732457E-01	-.4622987E-01
	B_6						.4738655E-01
THRUST CONTROL COEFFICIENTS	C_1	.5306218E+00	.762893E+00	.7719553E+00	.9994844E+00	1.0	1.0
	C_2						
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 if $V<V_c$, $\alpha=(62660.6/\sigma V^2)$ if $V>V_c$					
LAGRANGE MULTIPLIERS	ν_1	-.2631324E+00	.9737329E+00	.1166898E+01	.2256551E+01	.2280486E+01	.2265828E+01
	ν_2	-.3090828E+01	-.2866307E+01	-.2789790E+01	-.1896016E+01	-.1884320E+01	-.1879404E+01

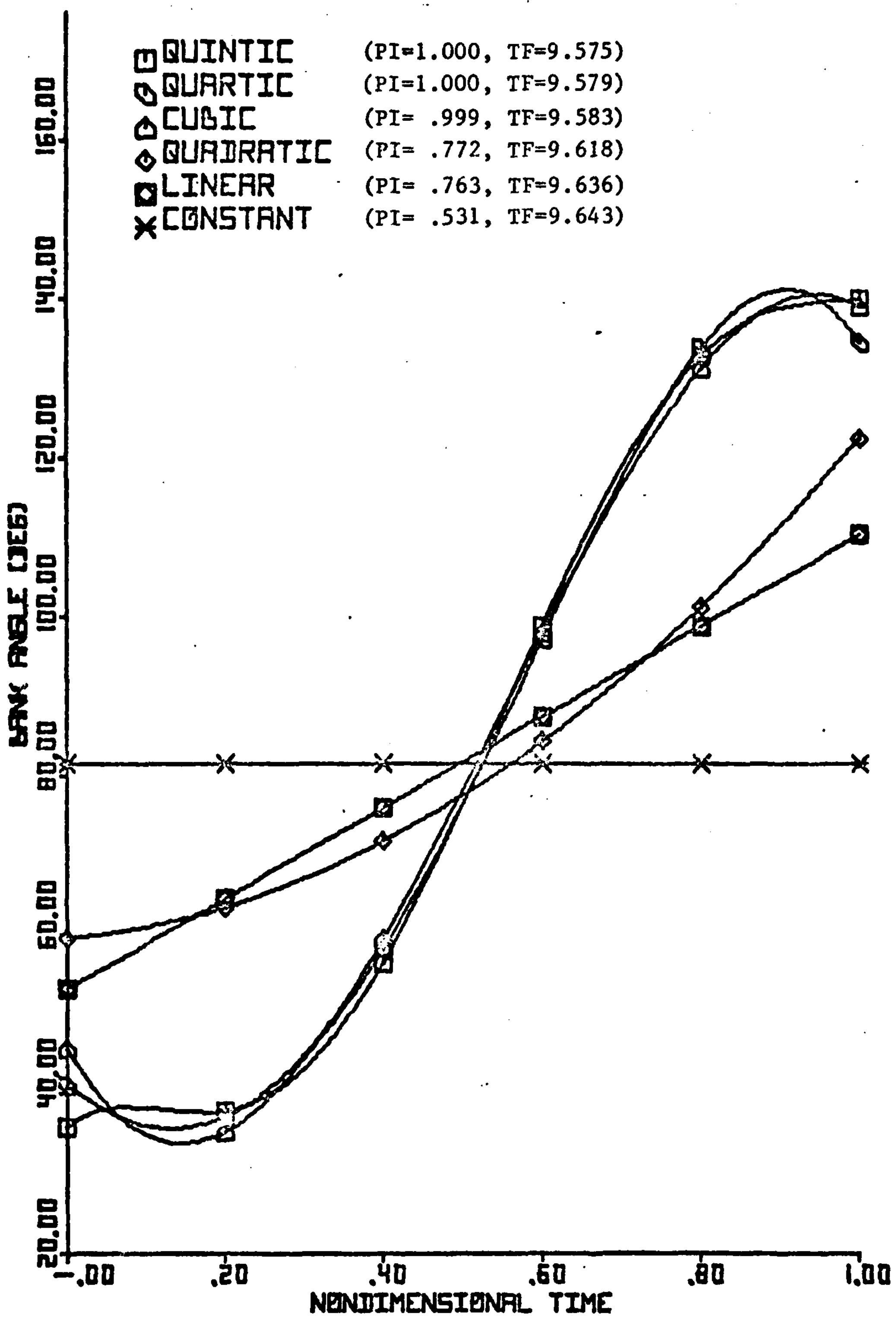

Fig. 2 Bank Angle Controls for Case 1

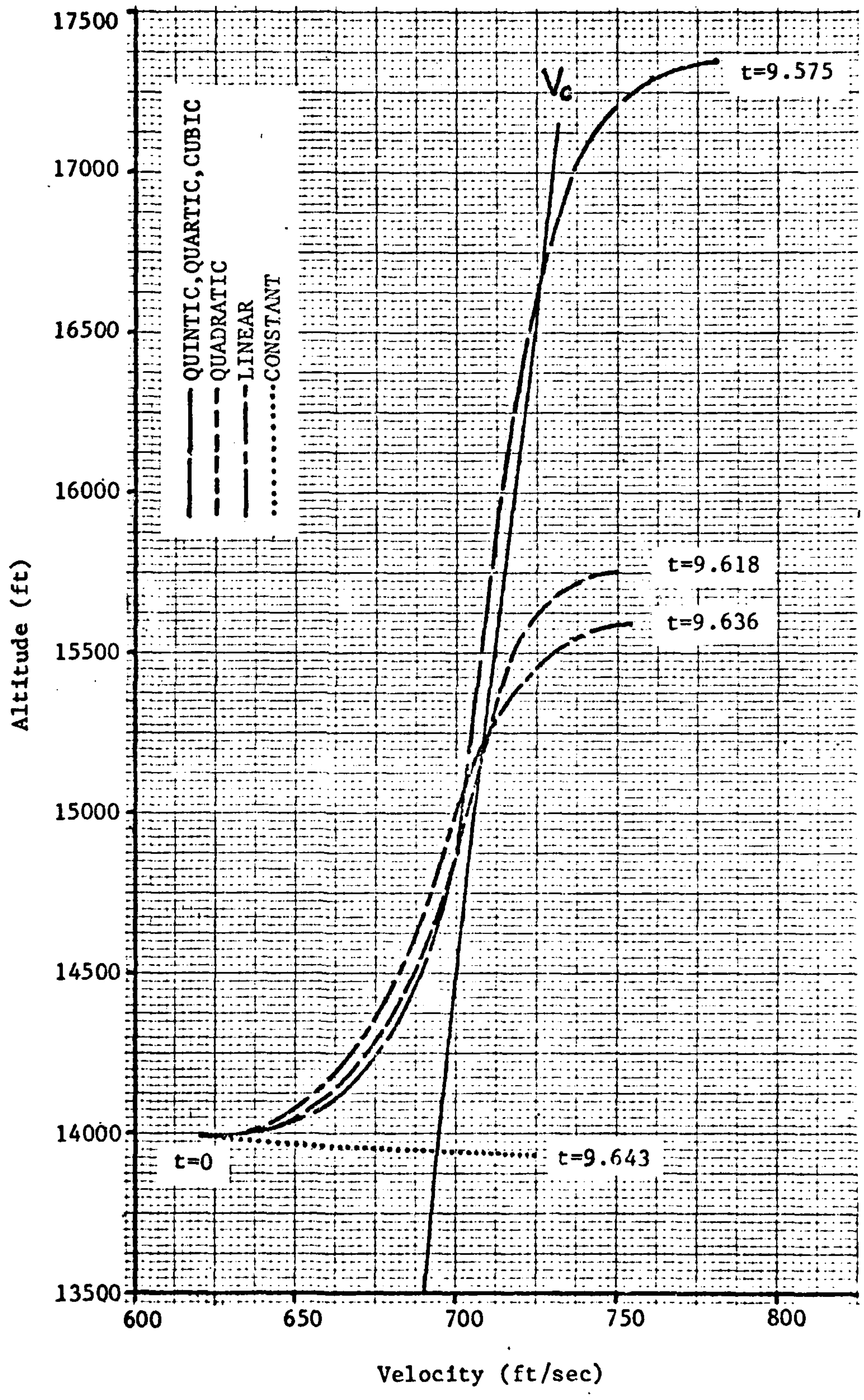

Fig. 3 Aircraft Trajectories for Case 1

Table 4

Optimal Coefficients for Case 2

	A	CONSTANT	LINEAR	QUADRATIC	CUBIC	QUARTIC	QUINTIC
MINIMUM TIME	t_f	.1117802E+02	.1083950E+02	.1083420E+02	.1083096E+02	.1083090E+02	
BANK ANGLE COEFFICIENTS	B_1	.1431889E+01	.1413989E+01	.1392642E+01	.1392420E+01	.1393745E+01	
	B_2		.3796622E+00	.3729148E+00	.3473183E+00	.3474240E+00	
	B_3			-.6128545E-01	-.6101403E-01	-.5815670E-01	
	B_4				-.4307462E-01	-.4317098E-01	
	B_5					.5783650E-02	
	B_6						
THRUST CONTROL COEFFICIENTS	C_1	0.0	0.0	0.0	0.0	0.0	
	C_2						
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 if $V<V_c$, $\alpha=(62[illegible]60.6/\sigma V^2)$ if $V>V_c$					
LAGRANGE MULTIPLIERS	ν_1	.3466188E+00	.3700110E+00	.2973495E+00	.2844843E+00	.2842218E+00	
	ν_2	-.3205328E+01	-.3184808E+01	-.3174672E+01	-.3177563E+01	-.3177074E+01	

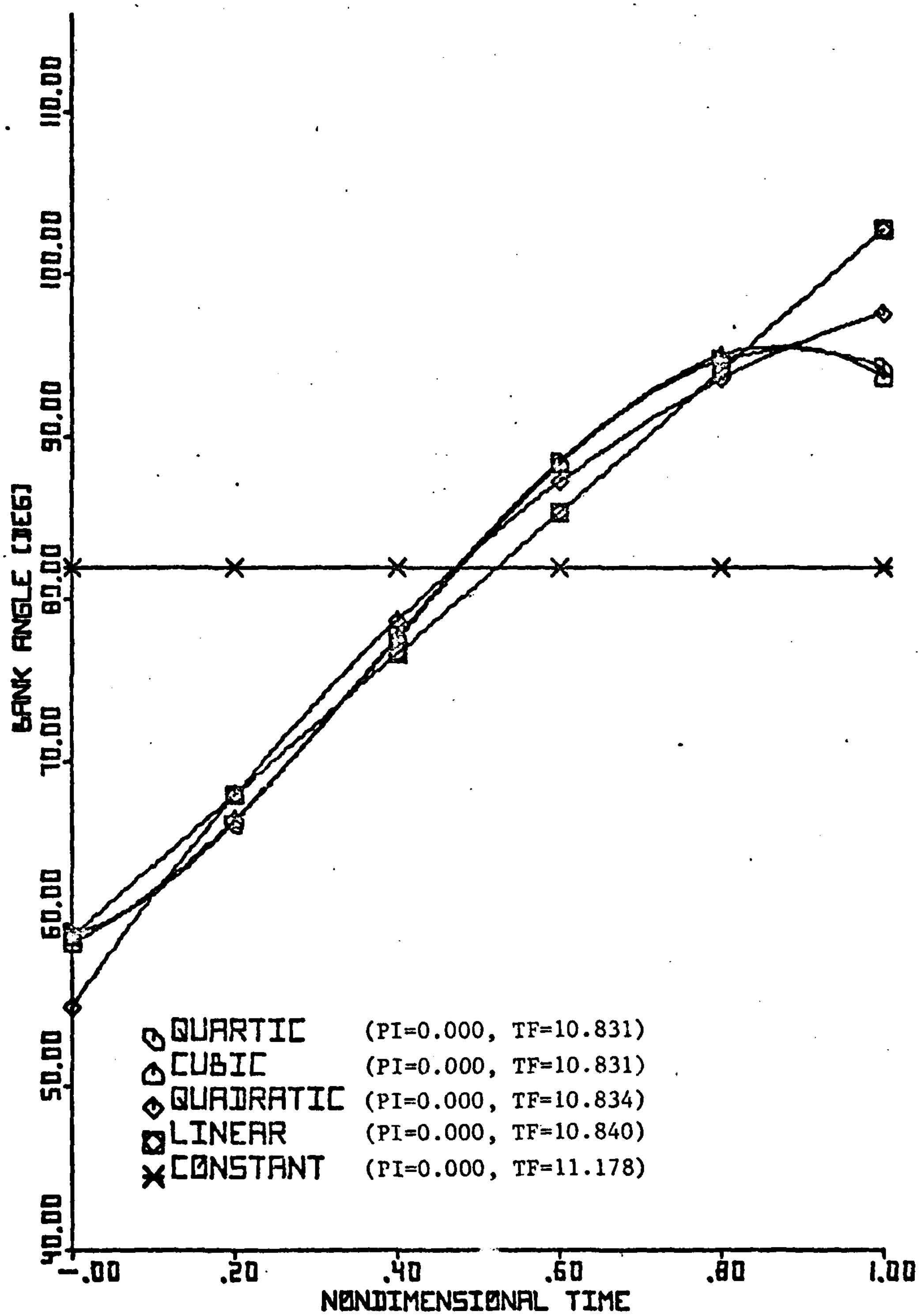

Fig. 4 Bank Angle Controls for Case 2

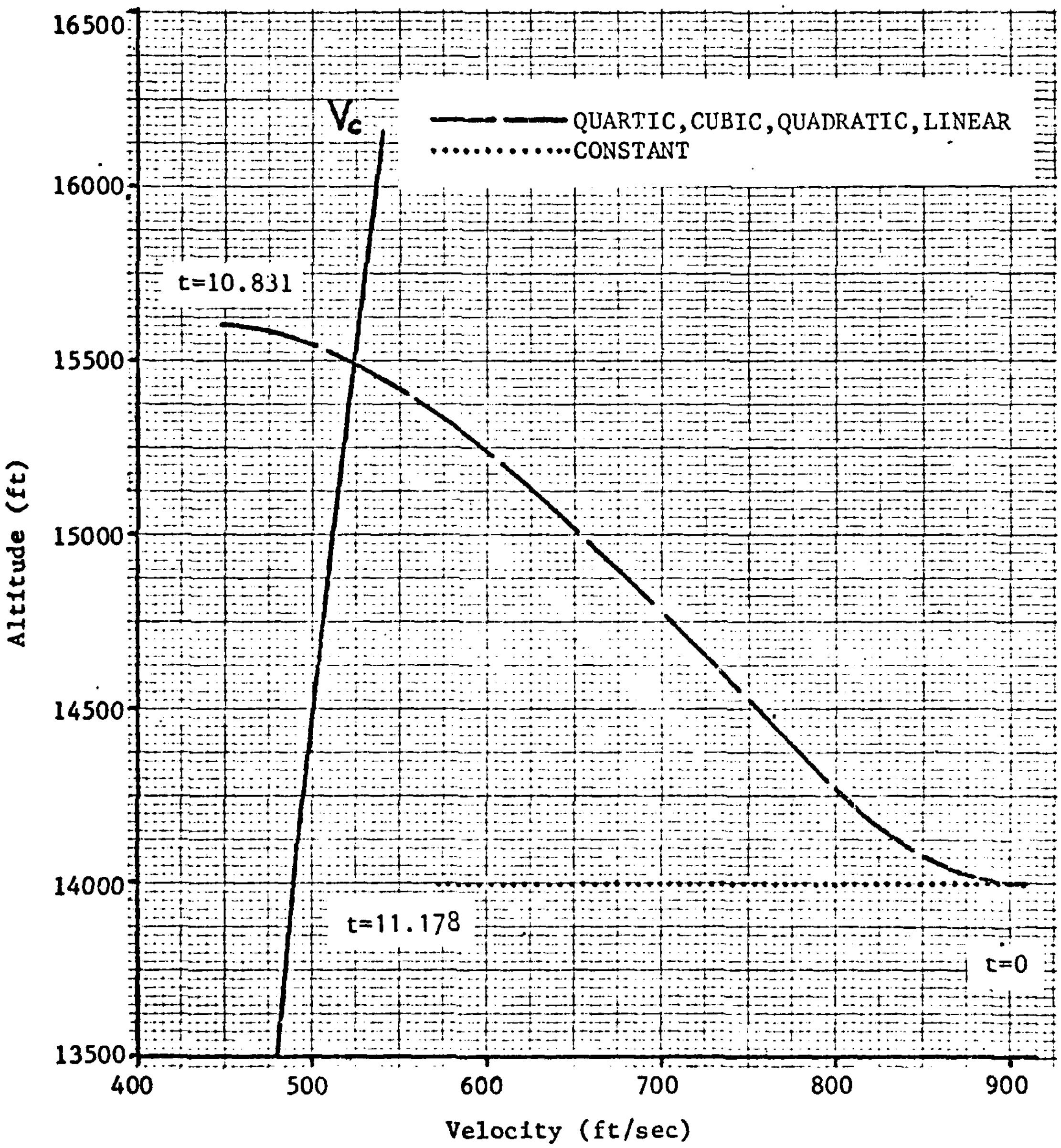

Fig. 5 Aircraft Trajectories for Case 2

Table 5

Optimal Coefficients for Case 3

	A	CONSTANT	LINEAR	QUADRATIC	CUBIC	QUARTIC	QUINTIC
MINIMUM TIME	t_f	.1071731E+02	.1057562E+02	.1057297E+02	.1052325E+02	.1052318E+02	
BANK ANGLE COEFFICIENTS	B_1	.1427728E+01	.1468263E+01	.1448469E+01	.1449513E+01	.1450784E+01	
	B_2		-.9350619E+00	-.9009187E+00	-.9245959E+00	-.9247146E+00	
	B_3			-.4369533E-01	-.6385080E-01	-.6048093E-01	
	B_4				.1858489E+00	.1847274E+00	
	B_5					.6473106E-02	
	B_6						
THRUST CONTROL COEFFICIENTS	C_1	-.2068547E+00	-.5749431E+00	-.5632677E+00	-.6	-.6	
	C_2						
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 if $V<V_c$, $\alpha=(62660.6/\sigma V^2)$ if $V>V_c$					
LAGRANGE MULTIPLIERS	ν_1	-.8532966E+00	-.2937694E+01	-.2928169E+01	-.3058386E+01	-.3061049E+01	
	ν_2	-.3353971E+01	-.2400173E+01	-.2440278E+01	-.2144212E+01	-.2144956E+01	

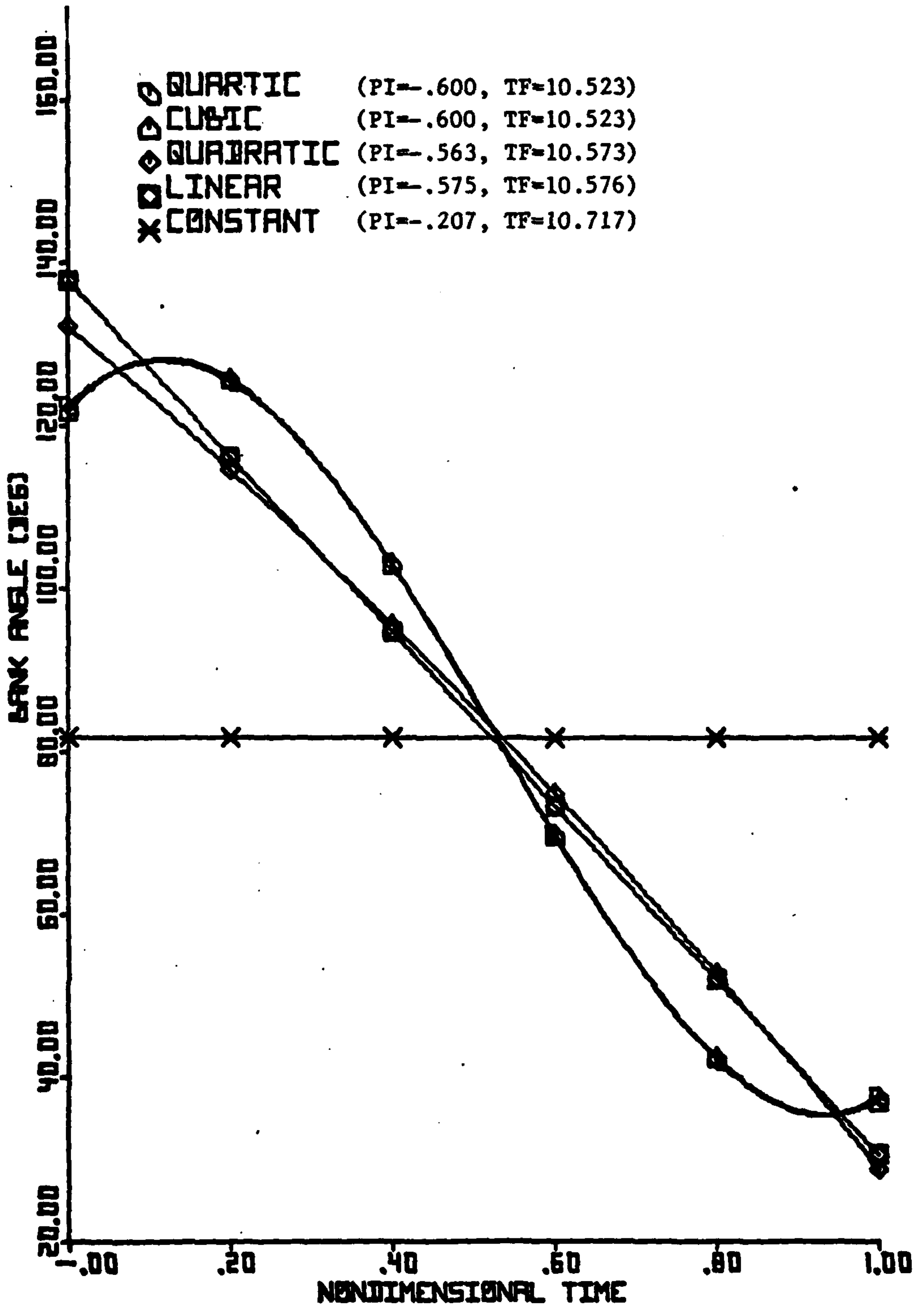

Fig. 6 Bank Angle Controls for Case 3

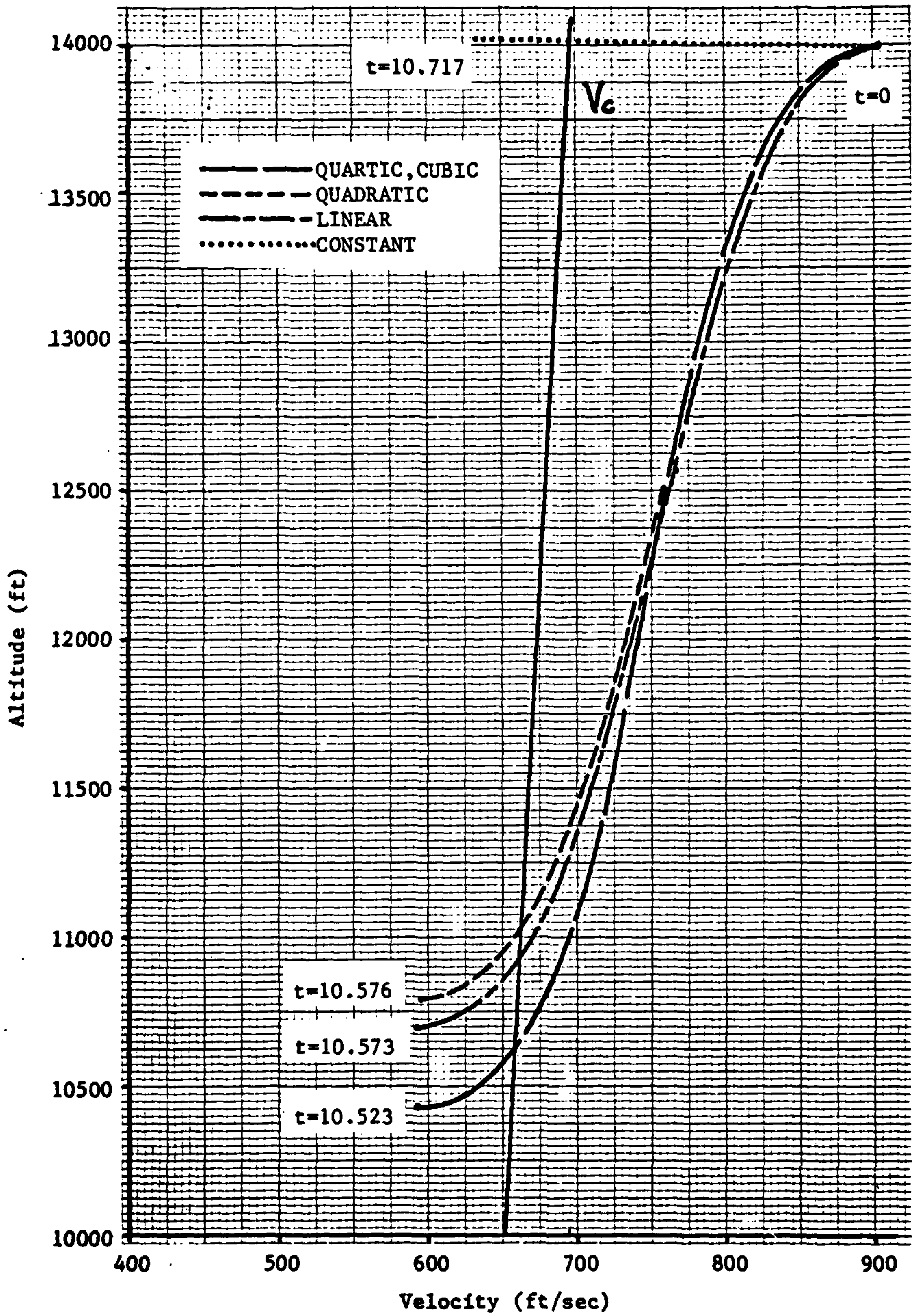

Fig. 7. Aircraft Trajectories for Case 3

Fig. 6, Fig. 7, respectively. In Figs. 2, 4, and 6, PI=π and TF= minimum final time . The times shown in Figs. 3, 5, and 7 are the minimum final times except for t=0 which identifies the initial conditions of the turn.

From Figs. 3, 5, and 7 it can be observed that the best turning times are obtained for cubic or higher order bank angle controls. In all cases, the trajectories for cubic or higher order bank angle controls are almost identical and cannot be distinguished individually upon the graphs. The turning times listed for those trajectories are the best times obtained. And also, the optimal constant thrust control variable values for those trajectories are equal to their maximum or minimum allowable values. The associated F_A terms for the thrust control variable coefficients indicated that the thrust control variable desired to exceed the allowable limits. Most important of all, it can be observed that better turning times are obtained when the aircraft's trajectories follow more closely to the V_c curve or when a larger portion of the trajectories are in the vicinity of the V_c curve. Based upon this observation, a higher order thrust control variable was used to attempt to reduce the turning times by maintaining the trajectories closer to the V_c curve since in all cases the trajectories approached the V_c curve, crossed it, and then continued <u>away</u> from it.

For Cases 4, 5, and 6, two different forms of a thrust control variable were used to improve turning times. First, a linear thrust control variable was used. Secondly, a bang-bang type thrust control variable was used since some of the linear results had a trend towards infinite slopes. A bang-bang thrust control is where the thrust varies instantaneously from its maximum or minimum allowable value to its minimum or maximum allowable value respectively. The time when the thrust control changes becomes a parameter in the problem. The results obtained using these two thrust control variable forms are combined with the best results obtained in Cases 1, 2, and 3 to form the total results in Cases 4, 5, and 6.

The results of Cases 4, 5, and 6 are listed and shown in Tables 6, 7, 8, Figs. 8, 10, 12, and Figs. 9, 11, and 13 respectively. Tables 6, 7, and 8 list the optimal coefficients for the various forms of controls used and the resulting Lagrange multipliers. Figs. 8, 10, and 12 show the optimal bank angle and thrust control variable (PI=π) histories. Figs. 9, 11, and 13 show the aircraft's trajectories for the various forms of the thrust control variable.

The results of Case 4 are best shown in Figs. 8 and 9. The optimal bank angle controls did not change appreciably for the three different forms for the thrust control variable. The three different associated aircraft trajectories varied only slightly near the later part of the trajectory. The minimum times to turn varied only slightly and the linear thrust control variable resulted in the best turning time. The final value for the linear thrust control variable is π=.1394 . This fact along with the fact that the bang-bang thrust control did not decrease the turning time indicates that thrust reversal would have no benefit in this case.

The results of Case 5 are best shown in Figs. 10 and 11. In this case, the optimal bank angle controls did change appreciably. The linear thrust control variable in this case tended to converge towards

Table 6

Optimal Coefficients for Case 4

	A	FULL ON THRUST	ON-OFF THRUST	ON-LINEAR THRUST
MINIMUM TIME	t_f	.9575270E+01	.9580989E+01	.9553958E+01
BANK ANGLE COEFFICIENTS	B_1	.1478629E+01	.1480379E+01	.1481717E+01
	B_2	.1066866E+01	.1021527E+01	.1022293E+01
	B_3	.1010573E+00	.1078232E+00	.1163696E+00
	B_4	-.2052865E+00	-.2287610E+00	-.2408131E+00
	B_5	-.4622987E-01	-.5017431E-01	-.3176035E-01
	B_6	.4738655E-01	.4920173E-01	.3962840E-01
THRUST CONTROL/ COEFFICIENTS	C_1	1.0	.8746054E+00	.1745161E+01
	C_2			-.1605740E+01
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 if $V<V_c$, $\alpha=(62660.6/\sigma V^2)$ if $V>V_c$		
LAGRANGE MULTIPLIERS	ν_1	.2265828E+01	.2138913E+01	.2129796E+01
	ν_2	-.1879404E+01	-.1925062E+01	-.1892610E+01

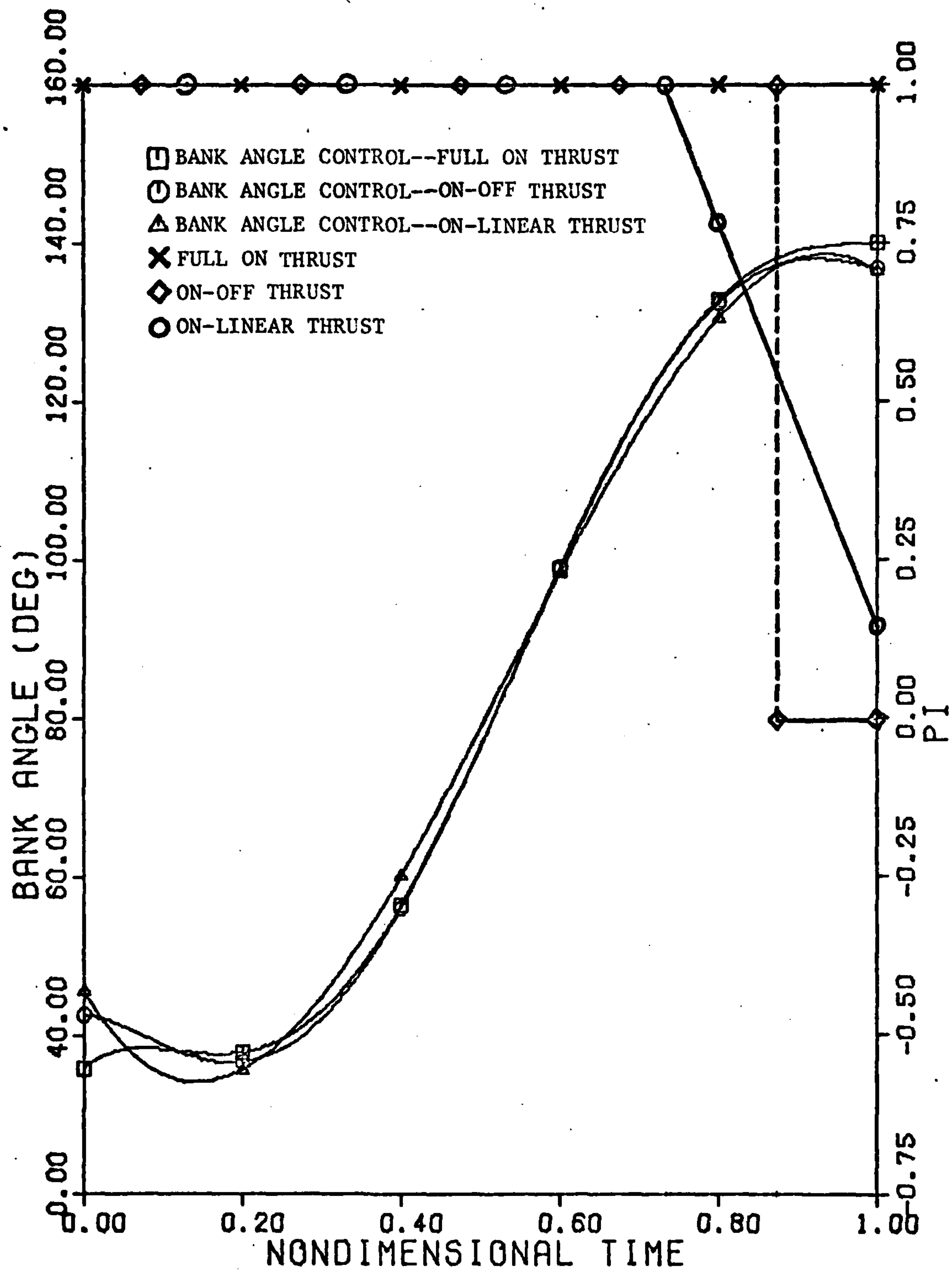

Fig. 8. Bank Angle and Thrust Controls for Case 4

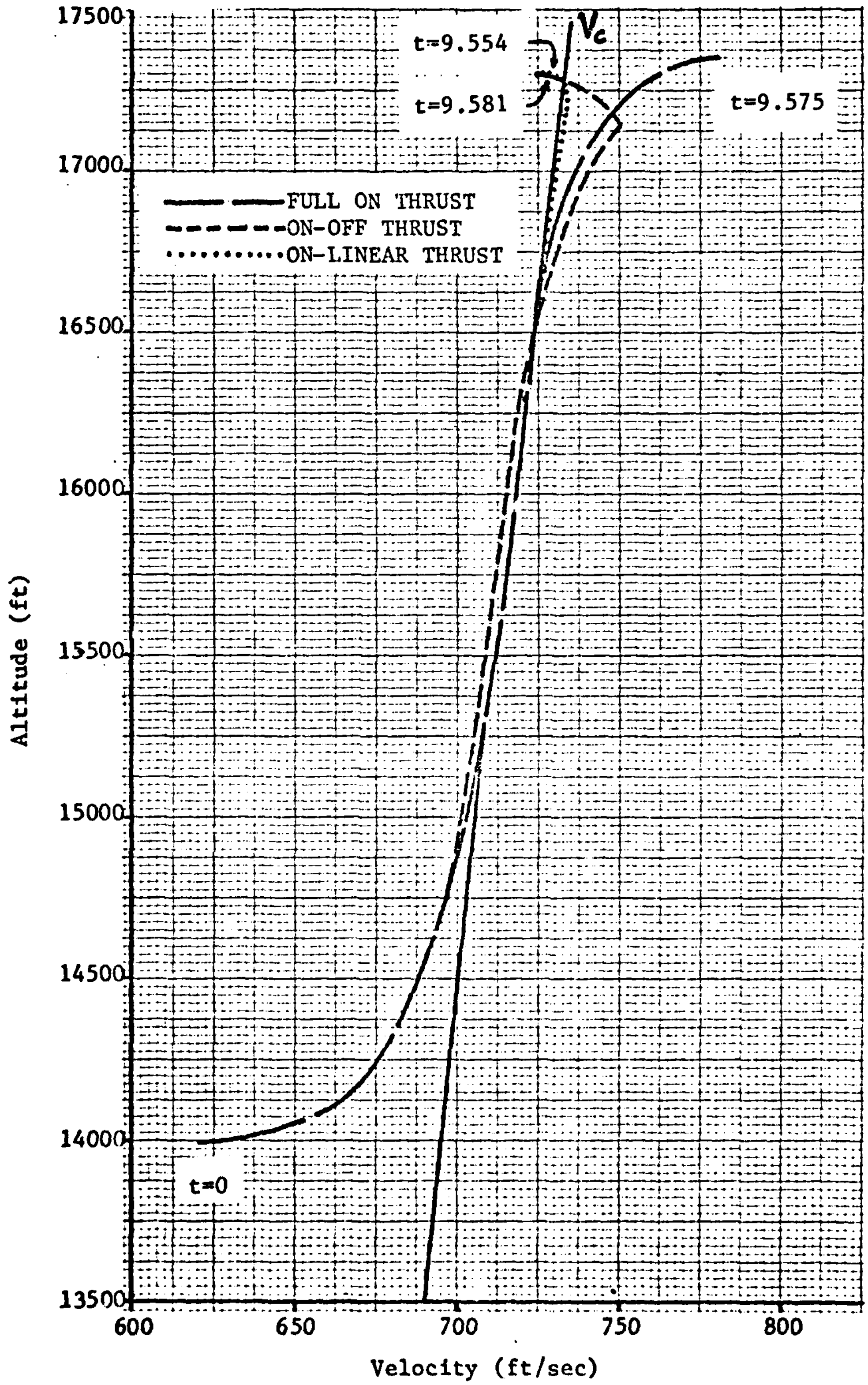

Fig. 9. Aircraft Trajectories for Case 4

Table 7

Optimal Coefficients for Case 5

	A	FULL-OFF THRUST	OFF-ON THRUST	OFF-LINEAR-ON THRUST
MINIMUM TIME	t_f	.1083090E+02	.1060527E+02	.1061267E+02
BANK ANGLE COEFFICIENTS	B_1	.1393745E+01	.1409545E+01	.1403790E+01
	B_2	.3474240E+00	.8437051E+00	.7815139E+00
	B_3	-.5815670E-01	.6862573E-01	.4517655E-01
	B_4	-.4317098E-01	-.1589530E+00	-.1198256E+00
	B_5	.5783650E-02	-.4818163E-01	-.4604624E-01
	B_6			
THRUST CONTROL COEFFICIENTS	C_1	0.0	.6368652E+00	-.5331165E+00
	C_2			.2799886E+01
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 if $V<V_c$, $\alpha=(62660.6/qV^2)$ if $V>V_c$		
LAGRANGE MULTIPLIERS	ν_1	.2842218E+00	.1893590E+01	.1666959E+01
	ν_2	-.3177074E+01	-.2350928E+01	-.2582316E+01

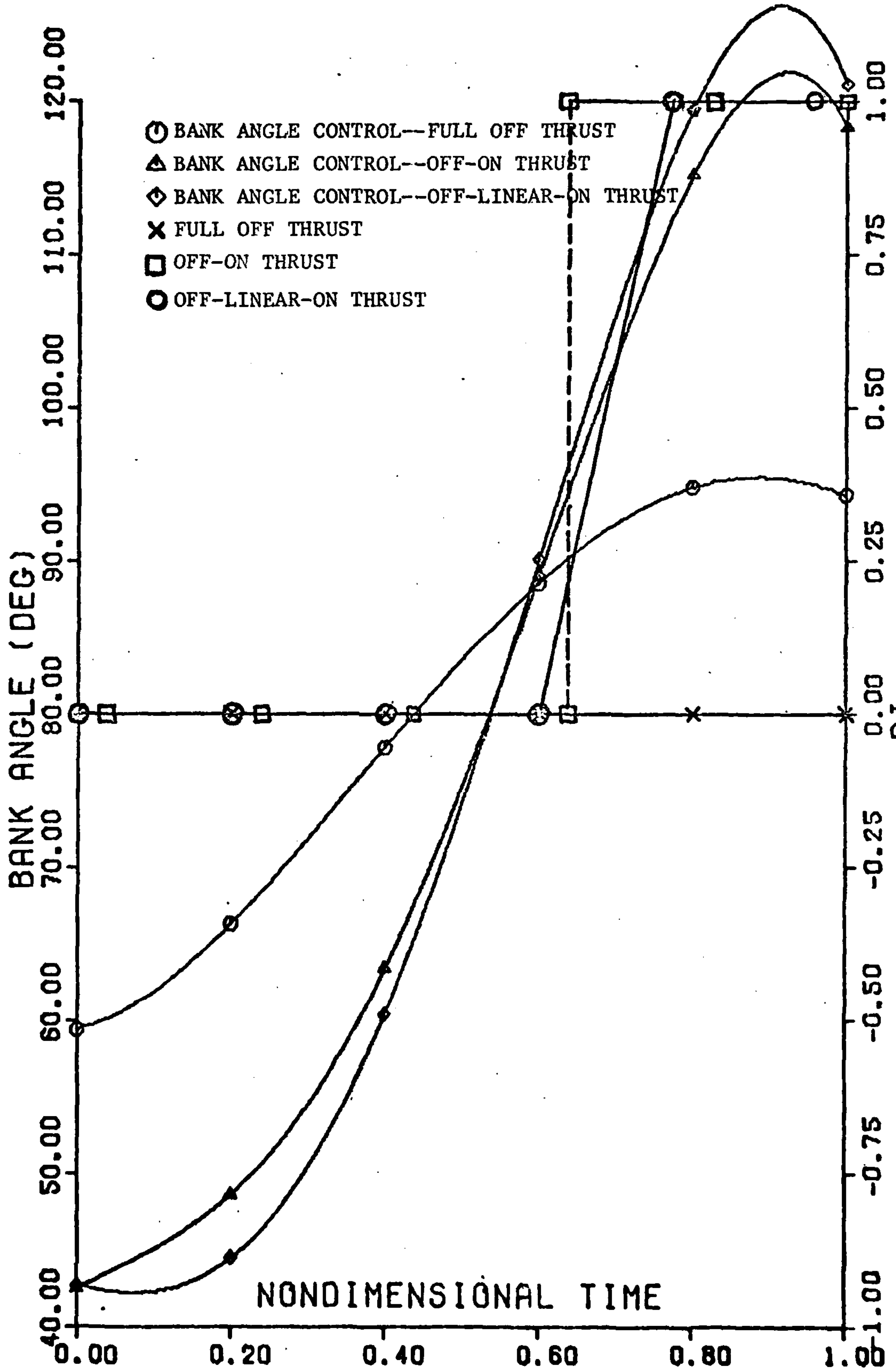

Fig. 10. Bank Angle and Thrust Controls for Case 5

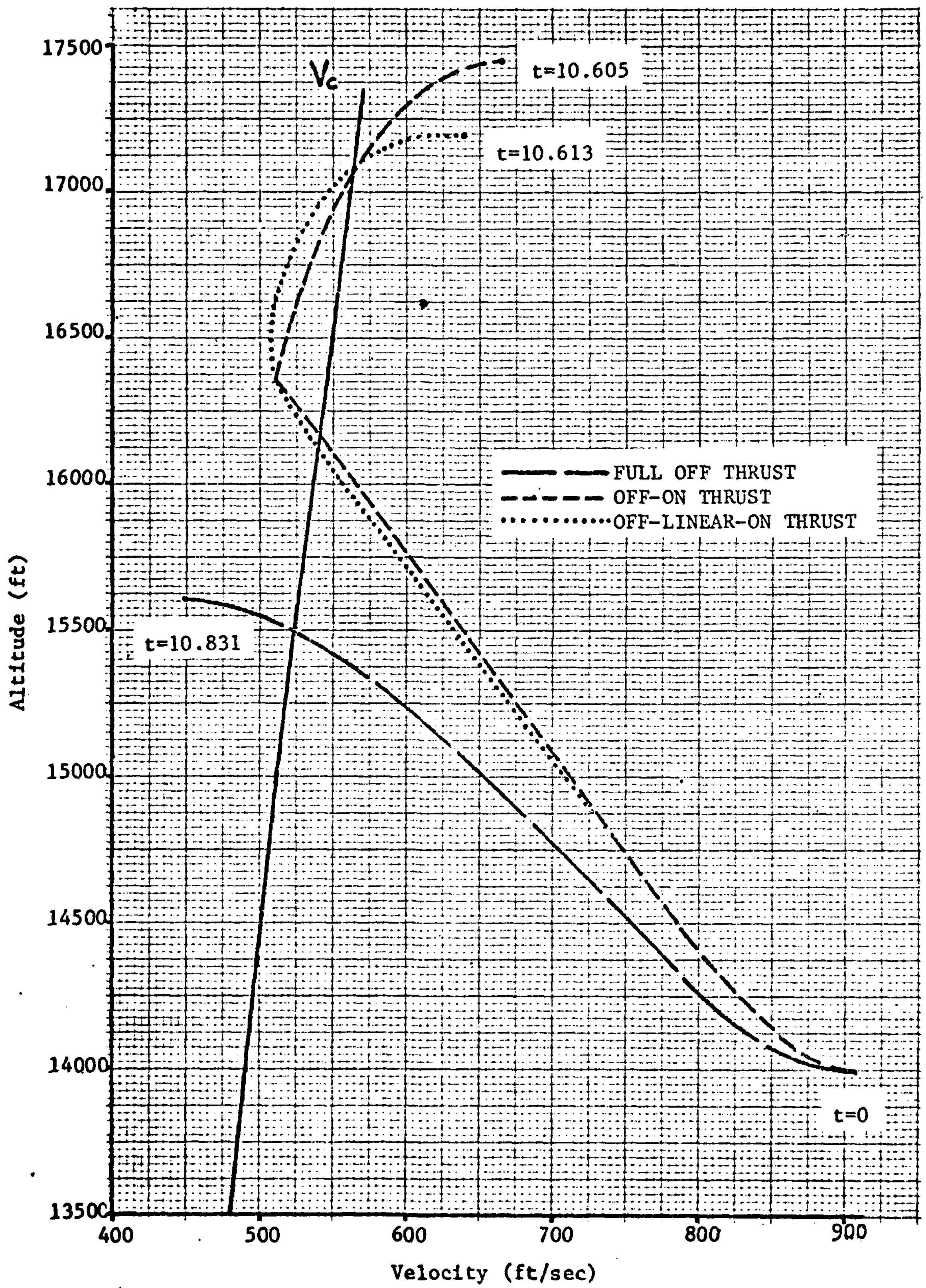

Fig. 11. Aircraft Trajectories for Case 5

Table 8

Optimal Coefficients for Case 6

	A	FULL REVERSE THRUST	REVERSE-ON THRUST	REVERSE-LIN-ON THRUST
MINIMUM TIME	t_f	.1052318E+02	.1025080E+02	.1025377E+02
BANK ANGLE COEFFICIENTS	B_1	.1450784E+01	.1445877E+01	.1444364E+01
	B_2	-.9247146E+00	-.4805901E+00	-.4001029E+00
	B_3	-.6048093E-01	-.3812296E-01	-.2616676E-01
	B_4	.1847274E+00	.9998157E-01	.8998715E-01
	B_5	.6473106E-02	.1030242E-01	.5692150E-02
	B_6			
THRUST CONTROL COEFFICIENTS	C_1	-.6	.7580790E+00	-.4445520E+02
	C_2			.1000517E+03
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 if $V<V_c$, $\alpha=(62660.6/qV^2)$ if $V>V_c$		
LAGRANGE MULTIPLIERS	ν_1	-.3061049E+01	-.1748365E+01	-.1474128E+01
	ν_2	-.2144956E+01	-.1765735E+01	-.2902723E+01

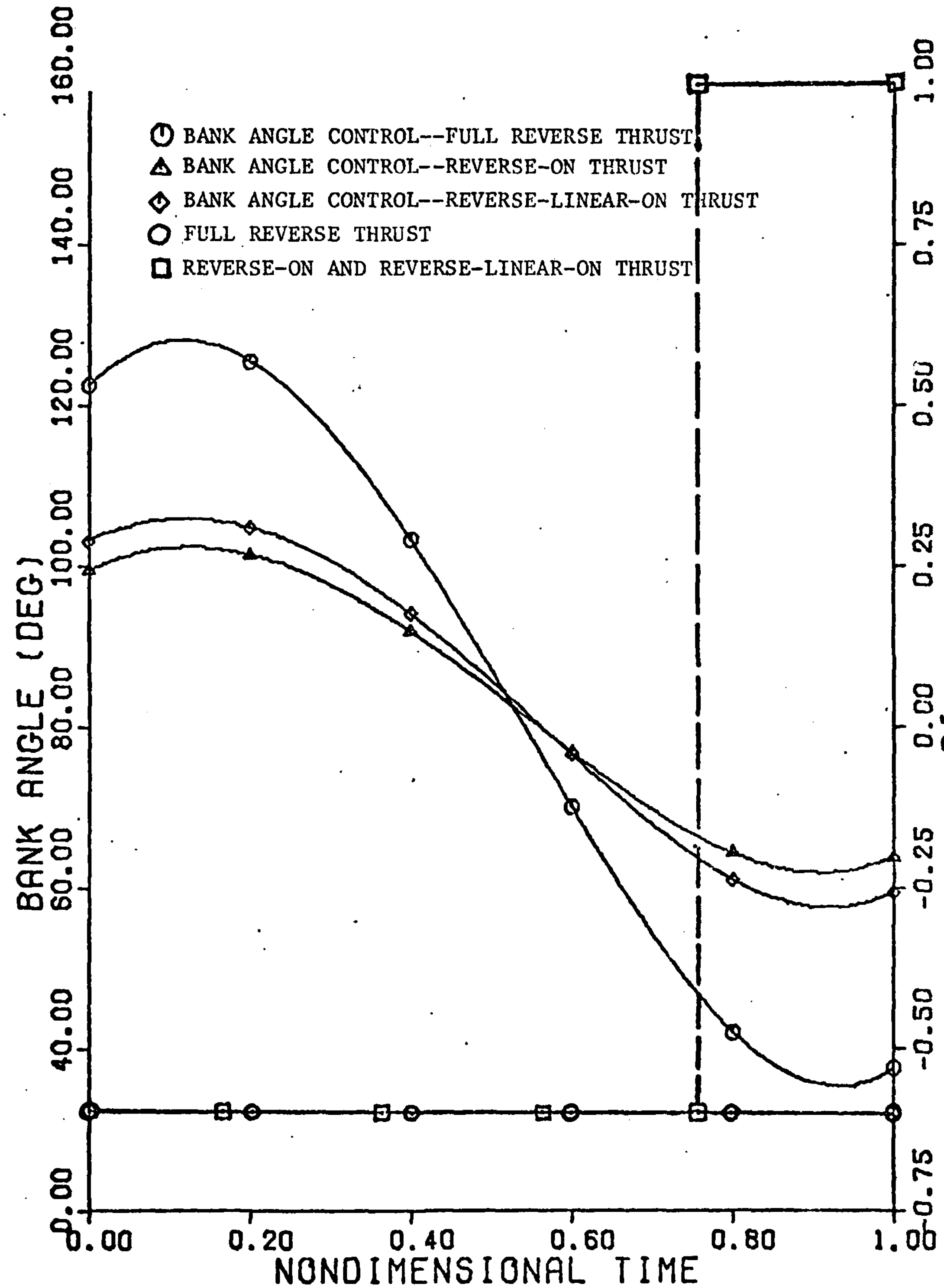

Fig. 12. Bank Angle and Thrust Control for Case 6

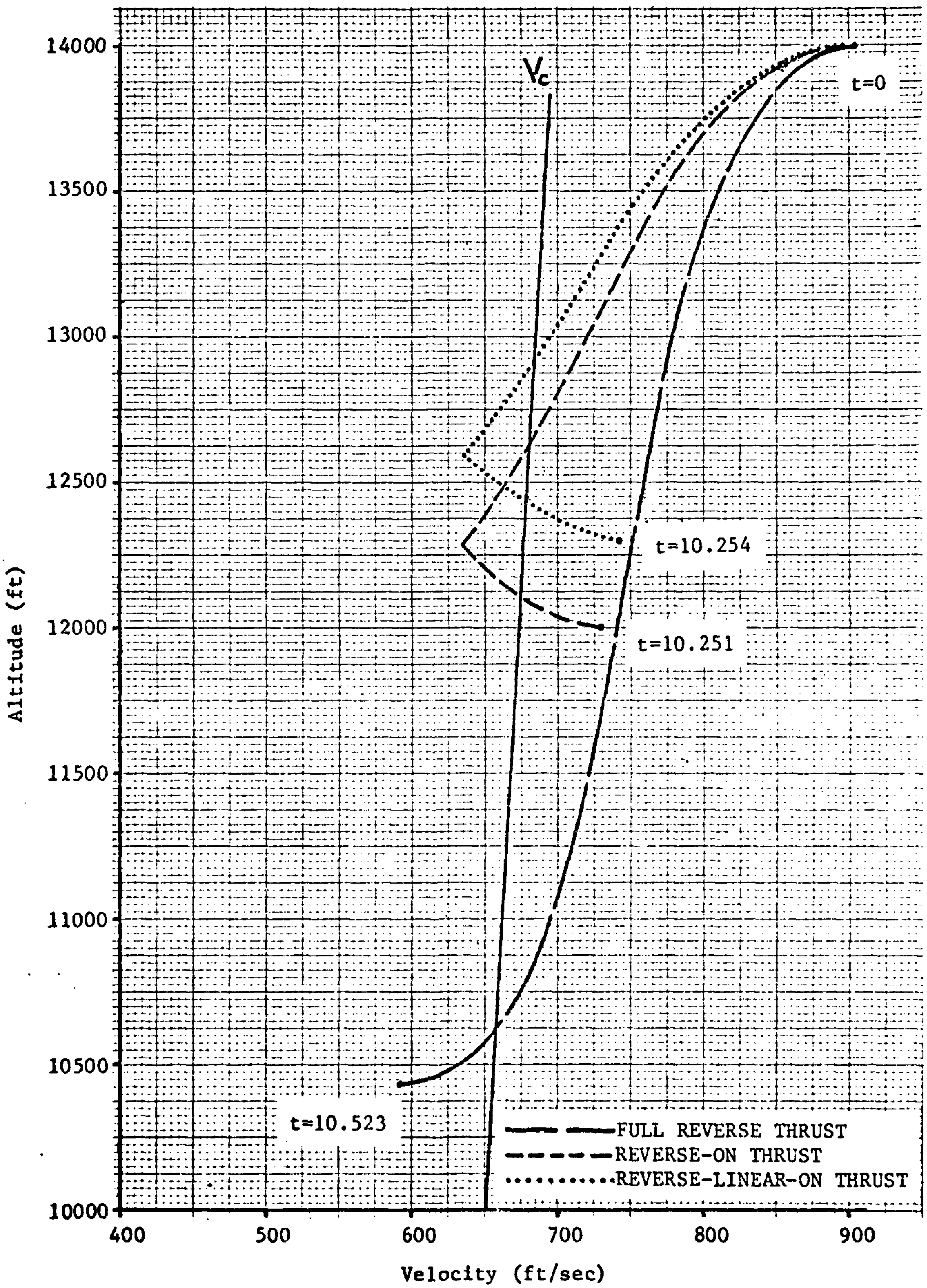

Fig. 13. Aircraft Trajectories for Case 6

an infinite slope solution. The off-linear-on thrust curve in Fig. 11 is the solution obtained prior to the program having difficulty in converging to an infinite slope solution. The bang-bang or off-on thrust control provided the best turning time. Note also the differences in the trajectories due to the different forms for the thrust control variable, and also the significant improvement in turning times.

The results of Case 6 are best shown in Figs. 12 and 13. Once again, results similar to Case 5 are obtained. There is an appreciable change in bank angle controls, trajectories, and turning times. Also, the linear thrust control variable tended to converge to an infinite slope solution and the reverse-linear-on thrust curve in Fig. 13 is the solution obtained prior to the program having difficulty in converging to an infinite slope solution.

No other forms for the thrust control variable were attempted since the results at this point gave a good indication of the minimum turning times and optimal trajectories that could be expected for the optimal solutions. In Case 4, the linear thrust control results in an optimal trajectory which follows the closest to the V_c curve. In Case 5, the bang-bang thrust control resulted in an optimal trajectory which followed more closely to the V_c curve for a greater part of the trajectory. The same took place in Case 6. These results can be improved upon. However, the change in turning times and the trajectories would not be significant as can be seen in Case 4. The benefit of thrust reversal can now adequately be evaluated by examining the minimum turning times and associated optimal trajectories.

In order to compare Case 1 and 2 to the results obtained in Ref (2) and to compare Case 4, 5, and 6 to evaluate the benefit of thrust reversal, Table 9 has been made. Table 9 contains information concerning the initial altitude and velocity, the final altitude and velocity, and the turning time of the best optimal trajectories in each of the cases. Also included is the initial and final specific energy of the aircraft for those trajectories. This is done because in an air-to-air combat situation it is desirable to always maneuver such that the aircraft's specific energy loss is minimized, since an advantage is maintained over an opponent by having more specific energy. Therefore, it is important to consider specific energy when evaluating thrust reversal. Specific energy can be determined from the expression

$$E = (h + \frac{V^2}{2g})\ \text{ft} \tag{104}$$

where $g=32.131\ \text{ft/sec}^2$. The best results of all cases are summarized in Table 9 as follows.

Table 9

Summary of Results

	h_i (ft)	V_i $(\frac{ft}{sec})$	h_f (ft)	V_f $(\frac{ft}{sec})$	E_i (ft)	E_f (ft)	E_f-E_i (ft)	t_f (sec)
DATA SET 6	13990	621	12300	794	19991	22110	2119	10.5
CASE 1	13990	621	17338	781	19991	26830	6839	9.575
DATA SET 12	13990	903	17634	886	26679	29850	3171	11.2
CASE 2	13990	903	15603	674	26679	22672	-4007	10.831
CASE 3	13990	903	10429	593	26679	15901	-10778	10.523
CASE 4	13990	621	17297	728	19991	25544	5553	9.554
CASE 5	13990	903	17421	783	26679	26961	282	10.605
CASE 6	13990	903	12004	729	26679	20274	-6405	10.251

VII Conclusions and Recommendations

The following conclusions are made based upon the results as summarized in Table 9.

1. The suboptimal control approach is an effective, credible, and easy means of finding optimal trajectories and minimum turning times.

2. Thrust reversal is not beneficial to reduce the minimum time to turn if the aircraft's initial velocity is below the corner velocity.

3. Thrust reversal is beneficial to reduce the minimum time to turn if the aircraft's initial velocity is above the corner velocity.

4. While thrust reversal is beneficial in reducing turning times, the loss of energy associated with these trajectories is too large a penalty to pay.

Using the suboptimal control approach, smaller minimum turning times for Case 1 and 2 were obtained than were obtained in Data Set 6 and 12 of Ref (2), respectively. Thus, the suboptimal control approach proved effective, credible, and more important, easier in finding optimal solutions. In Case 4, the best turning time was obtained using an on-linear thrust control which had a minimum value of π=.1394 at the final time (see Fig. 8). The corresponding optimal trajectory (see Fig. 9) followed the V_c curve the closest. Thus, thrust reversal would not be beneficial to reduce the turning time since it would not be used if the aircraft's initial velocity was below the corner velocity. However, in comparing Case 5 for the aircraft without thrust reversal and Case 6 for the aircraft with thrust reversal (see Table 9), thrust reversal does prove beneficial in reducing the minimum time to turn if the aircraft's initial velocity is above the corner velocity. Note that the difference in minimum turning times in these cases is .354 seconds. Thus, thrust reversal improves the minimum turning time by about 3.3%. In comparing Case 5 and 6 for the change in specific energy $(E_f - E_i)$, Case 5 does not lose energy, but gains energy slightly, and Case 6 loses a substantial amount of energy (see Table 9). Thus, thrust reversal is not beneficial in turning in minimal time without losing specific energy. Note that the difference in the change of specific energy in these two cases is 6,687 feet. Thus, thrust reversal increases the energy loss by 25.1%.

It is recommended that aircraft turns be studied from the point of view of reducing the energy loss during a turn. Future work could allow a certain fixed time, slightly greater than the minimum turning time, in which to perform a turn and find the optimal trajectories using various thrust limits that will minimize energy loss. Since thrust reversal in this study improved the minimum turning time by only 3.3% and increased the energy loss by 25.1%, it would be more important to minimize energy loss than to improve turning times in an air-to-air combat situation.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/GAE/AA-79D-6	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MINIMUM TIME TURNS WITH THRUST REVERSAL		5. TYPE OF REPORT & PERIOD COVERED MS Thesis
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Thomas L. Johnson Captain USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Institute of Technology (AFIT/ENA) Wright-Patterson AFB OH 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE December 1979
		13. NUMBER OF PAGES 75
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Approved for public release; IAW AFR 190-17

JOSEPH P. HIPPS, Major, USAF
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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Control
Optimization
Performance

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The object of this study is to find the optimal trajectories and corresponding minimum turning times for a high performance aircraft with and without thrust reversal to perform a prescribed turn, and then to compare those trajectories and times to evaluate the benefit of thrust reversal. Optimal control theory is applied to solve the minimum time to turn optimal control problem, using a suboptimal control problem approach and a second-order parameter-optimization method. The results of the study found that the suboptimal control approach was effective in solving the problem, that thrust reversal is beneficial in

reducing turning times if the aircraft's initial velocity is above the corner velocity, and that thrust reversal is not beneficial in performing a minimum time turn without losing energy.